

From Modern Physics
 by R. L. Sproull
 2nd Ed.
 John Wiley & Sons Publishers

12

Physical Electronics

12-1 INTRODUCTION

Physical electronics includes three different subjects: (1) The emission and absorption of electrons at the surfaces of solids, which will be considered in Secs. 12-2 to 12-7. (2) The collisions of electrons with atoms or ions in a gas, which will be considered in Sec. 12-8. (3) The trajectories of electrons and ions in electric and magnetic fields, which will be considered in Sec. 12-9. These rather different areas of physics are treated together because a mastery of all three is required to understand physical experiments or practical devices involving electron or ion beams. Now that the quantum physics has been developed in Chapters 5 to 10, we can return in this chapter to some of the phenomena discussed in an introductory manner in Chapter 4 and give a more thorough explanation of the basic physics involved.

The study of the quantum physics of solids in Chapters 8 to 10 provided the basis on which to discuss the emission of electrons from surfaces. The study of the quantum physics of atoms and molecules in Chapters 5 to 7 provided the basis on which to discuss collision processes. The physics of electron and ion trajectories is essentially part of classical physics, but interesting limitations are imposed on the classical theory by quantum physics.

12-2 THERMIONIC EMISSION

The emission of electrons by a hot metal or semiconductor is called thermionic emission. It is the principal practical source of electrons in commercial electron tubes and in laboratory experiments. The basic physics of thermionic emission is rather simple. An energy barrier a few electron volts in height at the surface of a solid prevents the emission of most of the electrons in the solid. At any temperature, however, some of the electrons have enough energy to surmount this barrier. The current of such electrons is a very sharply increasing function of T . The current from any known solid at room temperature is too small to be practically useful. Therefore practical emitters are always heated, usually to temperatures between 1000° and 2500°K .

In this section we shall first examine the nature of the surface-energy barrier. Then we shall calculate the thermionic-emission current from a metal and examine the effect of an applied electric field on the emission. Finally we shall investigate thermionic emission from semiconductors.

The potential-energy of an electron in a metal as a function of distance along a line of atom centers is illustrated in Fig. 12-1. This plot is like Fig. 8-9 except that the present sketch includes the surface, and therefore the regular array is not repeated to the right of the vertical line. There is no sharp definition of the exact position of the surface. Since there are no nuclei to the right of the vertical line, the potential energy curve does not turn downward at the right but approaches a horizontal asymptote, which represents the potential energy of an electron outside the metal.

The conduction band electron energies are also sketched on Fig. 12-1. The allowed energy band extends upward indefinitely, but at 0°K all the energy states with $E > E_0$ are empty, and at any temperature most

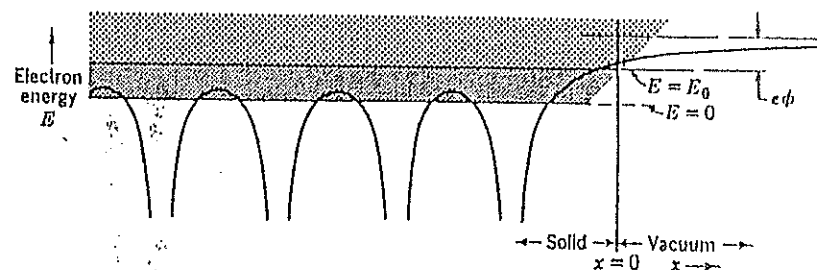


Fig. 12-1. Schematic diagram of the surface of a metal. An electron with energy $E_0 + e\phi$ inside the solid would have zero kinetic energy after emission.

of the states with $E \geq E_0$ are empty. It should be recalled that the kinetic energy of translation equals zero at the bottom of the conduction band, the point marked $E = 0$ in Fig. 12-1. The energy that must be given to an electron that initially had the energy E_0 in order to remove it from the solid is $e\phi$. This statement defines the work function $e\phi$ and is a refinement of the definition given in Sec. 1-2.

We next calculate the thermionic emission current density j from a metal at a temperature $T^\circ\text{K}$. The electrons emitted are the electrons with sufficient momentum normal to the surface to overcome the surface barrier. A necessary, but not sufficient, condition for emission of an electron is that its energy must be greater than $E_0 + e\phi$. This condition is not sufficient since the electron's momentum may not be directed toward the surface. If the momentum is directed away from the surface, of course the electron will not be emitted. If the momentum is directed generally toward the surface, but not perpendicular to the surface, the electron will be turned back from the surface unless it has an energy considerably greater than $E_0 + e\phi$. This situation is illustrated in Fig. 12-2. The surface force is in the $-x$ -direction and reduces the x component of momentum (p_x) of electrons striking the surface. If p_x is initially less than a critical value p_{xc} , the surface force reflects the electron back into the metal, regardless of the value of p_y . The energy $p_y^2/2m$ associated with the y component of momentum does not help the electron to surmount the barrier.

The critical value p_{xc} can be calculated by noting that the energy barrier has a height $E_0 + e\phi$; the difference in energy between an electron with zero kinetic energy inside the metal and an electron with zero kinetic energy outside the metal. Therefore the electron in order to escape must have at least the following value of the energy associated with motion in the x direction: $p_{xc}^2/2m = \frac{1}{2}mv_x^2 = E_0 + e\phi$. In other words,

$$p_{xc} = \sqrt{2m(E_0 + e\phi)} \quad (12-1)$$

Of course p_x must be positive.

The emission current per unit area is the product of the charge e and the number of electrons emitted per second per square meter. The number of electrons emitted is the product of v_x and the number per unit volume with $p_x > p_{xc}$. (This calculation of a flux or flow of particles in terms of v_x and the number per unit volume was explained in conjunction with Fig. 2-1; in the present calculation the factor of $\frac{1}{2}$ does not appear, since if $p_x > p_{xc}$ then p_x must be positive, and hence we do not include in the number per unit volume the electrons going toward $-x$.)

Therefore

$$j = ev_x \times (\text{Number}/\text{m}^3 \text{ with } p_x > p_{xc}) \quad (12-2)$$

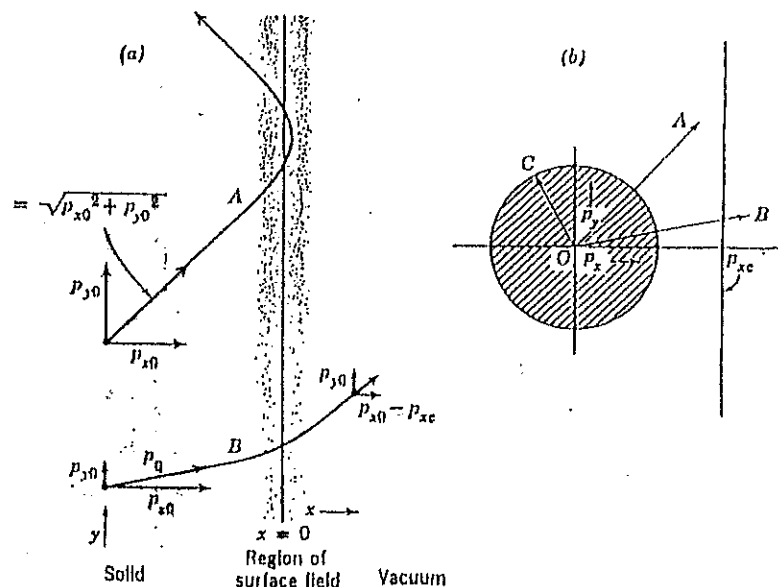


Fig. 12-2. (a) Two electron trajectories with the same initial energy ($> E_0 + e\phi$) inside the solid and therefore the same initial magnitude of momentum p_0 ; A is reflected by the surface field, and B is emitted. (b) The same situation as in (a) but plotted in momentum space instead of coordinate space. The vector terminating at C represents an electron with the energy E_0 . The vectors ending at A and B represent the reflected and emitted electrons, respectively, of (a).

We calculate the required density of electrons, as in Sec. 9-4, by multiplying the density of states $S(E)$ by the probability of occupation $f(E)$. Because of the condition on p_x we seek this density in terms of the momenta, rather than in terms of the energy as in Sec. 9-4. The number of traveling-wave states per cubic meter with p_x between p_x and $p_x + dp_x$, and similarly for p_y and p_z , is

$$(2/h^3) dp_x dp_y dp_z \quad (12-3)$$

from eq. G-13 of Appendix G. The fraction of these that are occupied is given by the Fermi function (eq. 9-4). But our calculation need be valid only for $E \gg E_0$, since the work function $e\phi$ is of the order of 1 volt or greater. Therefore we can use the approximate form of the Fermi function, eq. 9-2:

$$f(E) = e^{-(E-E_0)/kT} = e^{-(p_x^2 + p_y^2 + p_z^2 - 2mE_0)/2mkT} \quad (12-4)$$

We can now insert eqs. 12-3 and 12-4 into eq. 12-2 and use the fact that $mv_x = p_x$:

$$j = \frac{2e}{mh^3} \int_{p_y=-\infty}^{\infty} \int_{p_z=-\infty}^{\infty} \int_{p_x=p_{xc}}^{\infty} p_x dp_x dp_y dp_z e^{-\left(\frac{p_x^2 + p_y^2 + p_z^2 - 2mE_0}{2mkT}\right)}$$

$$= \frac{2e}{mh^3} \int_{-\infty}^{\infty} e^{-\frac{p_y^2}{2mkT}} dp_y \int_{-\infty}^{\infty} e^{-\frac{p_z^2}{2mkT}} dp_z \int_{p_{xc}}^{\infty} e^{-\left(\frac{p_x^2 - 2mE_0}{2mkT}\right)} p_x dp_x \quad (12-5)$$

The first two integrals can be evaluated by using the fact that $\int_{-\infty}^{\infty} e^{-ux^2} = (\pi/u)^{1/2}$, and each integral has the value $(2\pi mkT)^{1/2}$. The third integral can be evaluated by inserting p_{xc} from eq. 12-1 and substituting u for the function in the exponent:

$$\int_{\sqrt{\frac{2m(E_0 + e\phi)}}}^{\infty} e^{-\left(\frac{p_x^2 - 2mE_0}{2mkT}\right)} p_x dp_x = mkT \int_{\phi/kT}^{\infty} e^{-u} du = mkT e^{-e\phi/kT}$$

If we put these evaluations into eq. 12-5 we obtain

$$j = \left(\frac{4\pi m e c k^2}{h^3}\right) T^2 e^{-e\phi/kT} = A_0 T^2 e^{-e\phi/kT} \quad \text{amp/m}^2 \quad (12-6)$$

This is the Richardson-Dushman thermionic-emission equation. A_0 is a universal constant, equals 1.20×10^6 amp/m² deg². We shall defer comparison of this equation with experiment until after the discussion of a modification of it.

The effect on j of the application of an electric field to the surface of the metal will now be considered. In order to do this it is necessary to look more closely at the nature of the force on the emitted electron as a function of its distance x from the surface of the metal. It is fortunate that we need to know the force only some distance away from the surface ($x = 0$), since near $x = 0$ the situation is complicated by the crystal structure and the atomic electron distributions (see prob. 12-7). The force between the electron and the metal can be computed from ordinary electrostatics when an electron is many lattice constants away from the surface. Under these conditions, the metal surface in Fig. 12-1 can be assumed to be plane and continuous, as shown in Fig. 12-3a.

All the electric field lines must intersect the surface at right angles, since the metal is a conductor. Evidently it would be tedious to calculate the positive charge on the surface as a function of position and then to calculate the force exerted on the electron by this charge distribution. There is a short cut in this electrostatic problem that is based upon the method of images. The field to the right of $x = 0$ is identical in Figs.

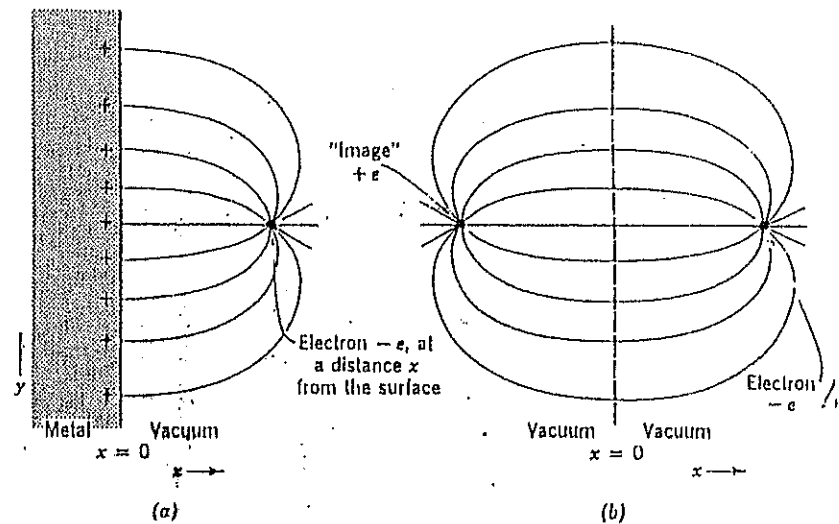


Fig. 12-3. (a) Electric field lines for an electron near the surface of a metal. (b) Electric field lines for an electron and a charge $+e$ at equal distances either side of $x = 0$; the field for $x > 0$ is identical with the field in (a), since the field lines in (a) are perpendicular to the metal surface.

12-3a and 12-3b. This means that the force on the electron at a distance x from the surface is the same as if the metal surface were replaced by a charge $+e$ at $-x$. The force on the electron is therefore $-e^2/(16\pi\epsilon_0 x^2)$, since the distance from the electron to its image is $2x$.

The potential energy associated with this force is $-e^2/(16\pi\epsilon_0 x)$, computed on the assumption that $P = 0$ at $x = \infty$. If there is no external electric field applied to the emitting surface, the only contribution to P is the image force contribution, and this term is illustrated in Fig. 12-1a. If an accelerating field \mathcal{E} is applied, the situation illustrated in Fig. 12-1b prevails. There is now an additional contribution $-e\mathcal{E}x$ to the potential energy of an electron. The electron needs somewhat less ($e\Delta\phi$) than an energy $E_0 + e\phi$ in order to be emitted. Therefore more electrons will surmount the barrier, and the resulting increase in j is called the Schottky effect.

In order to calculate this increase we first calculate x_0 , the position of the maximum of $P(x)$ in Fig. 12-1b. Since the total potential energy of the electron is $P = (-e^2/16\pi\epsilon_0 x) - \mathcal{E}x$, we can locate the position x_0 of the maximum by setting $dP/dx = 0$:

$$\frac{e^2}{16\pi\epsilon_0 x_0^2} - \mathcal{E}e = 0 \quad x_0 = \left(\frac{e}{16\pi\epsilon_0 \mathcal{E}}\right)^{1/2} \quad (12-7)$$

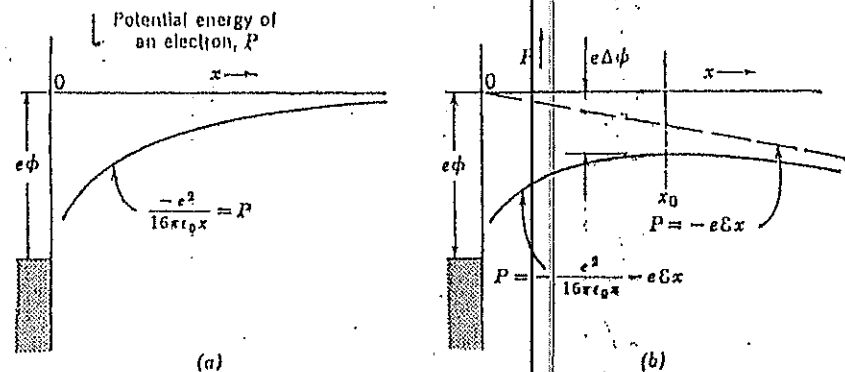


FIG. 12-4. Potential energy P of an electron as a function of its distance x from the surface of a metal. (a) No external field. (b) External field accelerating electrons toward $+x$; a lowering $\Delta\phi$ of the work function has been produced (ϵ and $\Delta\phi$ are greatly exaggerated).

We can now calculate $e\Delta\phi$, since this is the value of P at $x = x_0$:

$$e\Delta\phi = \frac{-2e^{1/2}\epsilon^{1/2}}{(16\pi\epsilon_0)^{1/2}} \quad \Delta\phi = -\left(\frac{e\phi}{4\pi\epsilon_0}\right)^{1/2} \quad (12-8)$$

This $e\Delta\phi$ is the amount by which the work function $e\phi$ is lowered when an accelerating field ϵ is applied. The thermionic emission equation including the effect of the applied electric field is therefore

$$j = A_0 T^2 e^{-\frac{e}{kT} \left[\phi - \left(\frac{e\phi}{4\pi\epsilon_0} \right)^{1/2} \right]} \quad (12-9)$$

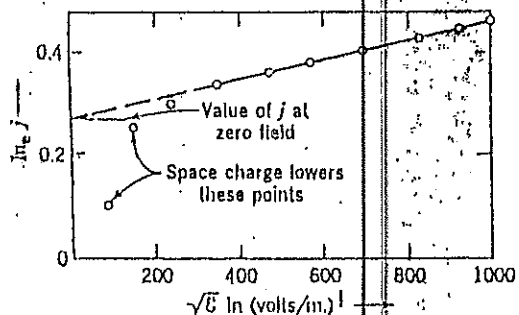


FIG. 12-5. Schottky effect with a tungsten filament at 2100°K . The current at zero field is about 0.8 of the current at 10^6 volts per meter. The units of j and the vertical position of the line (the "Schottky line") are arbitrary.

At a constant temperature, a plot of $\ln_e j$ vs. $\sqrt{\epsilon}$ should be a straight line. This result agrees well with experiment, as illustrated in Fig. 12-5, which is a plot of the logarithm of the cathode current density in a diode as a function of the square root of the electric field at the cathode (proportional to the square root of the anode voltage). The current at very low anode voltages is limited by space charge; much of the emitted current is turned back at the potential minimum (potential energy maximum) outside the cathode created by the space charge. For higher values of ϵ , the Schottky-effect equation (eq. 12-9) is well verified. This agreement enables us to extrapolate the Schottky line (Fig. 12-5) back to $\epsilon = 0$ in order to obtain the zero field emission current density, which is the j predicted by eq. 12-6.

Comparison of eq. 12-6 with experiment is shown in Fig. 12-6. The natural logarithm of the zero-field j divided by T^2 is plotted as a function of $10,000/T$, and a straight line is obtained. From the slope and

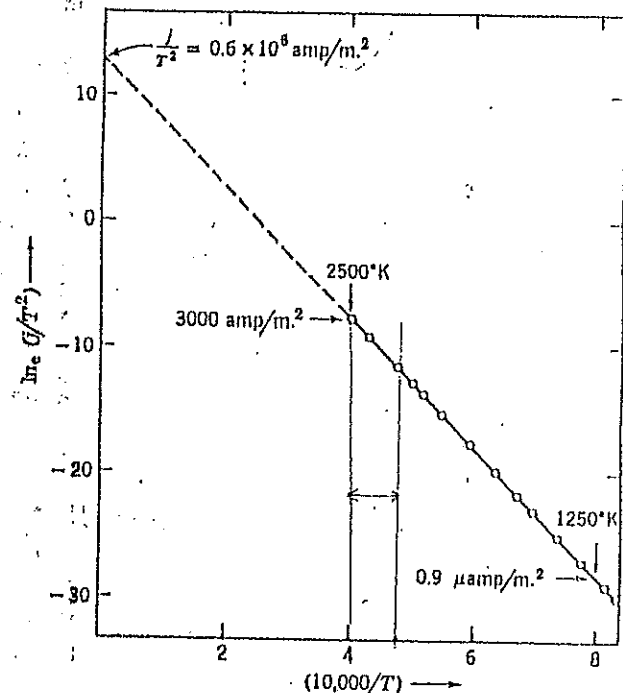


FIG. 12-6. Thermionic emission from tungsten. The ordinate is the logarithm to the base e of the current density (amp/m.²) divided by T^2 . The A factor can be determined from the $(10,000/T) = 0$ intercept, and the work function can be determined from the slope.

TABLE 12-1

Thermionic Emission Constants

Metal	ϕ , volts	A , amp/m. ²	Metal	ϕ , volts	A , amp/m. ²
Cr	4.60	0.48×10^6	Pt	5.32	0.32×10^6
Fe	4.48	0.20	Ta	4.10	0.55
Mn	4.20	0.55	W	4.62	0.80
Ni	4.61	0.30	LaB ₆	2.06	0.20

From C. Herring and M. H. Nichols, *Revs. Mod. Phys.*, 21, 185-270 (1949), and J. M. Lafferty, *J. Appl. Phys.*, 22, 200 (1951).

($10,000/T$) = 0 intercept of such a line we can obtain the work function ϕ and A -factor (the experimental value of the quantity A_0 in eq. 12-6) for various materials. Some experimental values are tabulated in Table 12-1. It is very difficult to calculate ϕ values from the quantum theory of metals: it has been done only for the alkali metals. These metals melt at too low temperatures to permit thermionic determinations of ϕ , but the theory is in good agreement with photoelectric determinations of ϕ .

The A values in Table 12-1 can be compared with the theoretical value of 1.20×10^6 amp/m.². The experimental determinations are not accurate because of the large extrapolation of data required (see Fig. 12-6 and prob. 12-6), but the disagreements are larger than the experimental error. There are two principal causes for this disagreement:

1. We have tacitly assumed in analyzing the data of Fig. 12-6 that ϕ is a constant independent of temperature. ϕ should not be exactly constant, since the expansion of the solid with temperature and other, more minor effects change ϕ slowly with T . Because this variation is small compared to ϕ itself, it is possible to use a Maclaurin expansion and to write $\phi = \phi_0 + T(d\phi/dT)_{T=0}$ to a good approximation. If this ϕ is substituted into eq. 12-6, we obtain

$$j = [A_0 e^{-\frac{\phi_0}{kT}} \left(\frac{d\phi}{dT} \right)_{T=0} T^2] e^{-\frac{\phi_0}{kT}} \quad (12-10)$$

The quantity in brackets is the predicted A factor and can be considerably different from A_0 .

2. We have tacitly assumed a perfectly uniform surface of the emitter with the same ϕ at all positions, but actually ϕ is different for different crystal faces of the emitter, and the experiments are performed on poly-

crystalline wires. Suppose that half the emitter area has a work function ϕ_1 and half has ϕ_2 , where ϕ_2 is considerably greater than ϕ_1 . Practically all the emission is from the patches with the lower work function ϕ_1 . The emitting area is therefore only half the nominal cathode area, and the measured A will be one-half the theoretical A_0 for a uniform emitter. This patch effect also complicates the Schottky effect, since there are strong local electric fields between adjacent patches.

Both these effects are probably important in the measurements summarized in Table 12-1. Another possible effect is probably not of much practical importance but deserves brief mention. This is the partial reflection of the electron waves as they emerge from the surface. The reflection effect was described in Sec. 5-1d; it can be serious for a sharp step in the potential energy of an electron. But the reflection coefficient r is far less than unity if the potential energy changes slowly with distance, as required by the image force theory. The factor $(1 - r)$ really should multiply the right side of eqs. 12-6, 12-9, and 12-10, but is very nearly equal to unity. There are nevertheless small but interesting interference effects between the partial reflections at $x = 0$ and at $x = x_0$ that produce a periodic deviation of experimental data from the Schottky line.

Pure tungsten is the only one of the three kinds of practical thermionic emitters to which the theory just presented is directly applicable. Tungsten is a useful emitter because it has the highest melting point and lowest vaporization rate at high temperatures of any metal. A tungsten filament is ordinarily operated at a temperature of about 2500°K. A higher temperature would give greater emission current density (j), and would even give a greater j per watt of filament heating power, but higher temperatures would reduce the life of the filament because of evaporation. At $T = 2500^\circ\text{K}$ the thermionic emission current density from tungsten is about 3000 amp/m.². Because of the low efficacy (low j per watt of heating power), pure tungsten emitters are employed only where other emitters cannot be used. Their application is therefore confined to power tubes (transmitting tubes) and X-ray tubes, where the anode voltage is so high that energetic positive ions can be produced that damage other kinds of emitters.

The second kind of practical emitter is the thoriated-tungsten emitter, which is a filament of tungsten containing a few per cent of ThO₂. The filament must be activated in a vacuum by the following treatment: First it is heated to 2800°K for a few minutes to clean the surface by evaporation and to reduce some ThO₂ to Th. Then it is cooled to about 2100°K. At this temperature more thorium diffuses to the surface than evaporates, and thus a thorium coating is created. This coating is