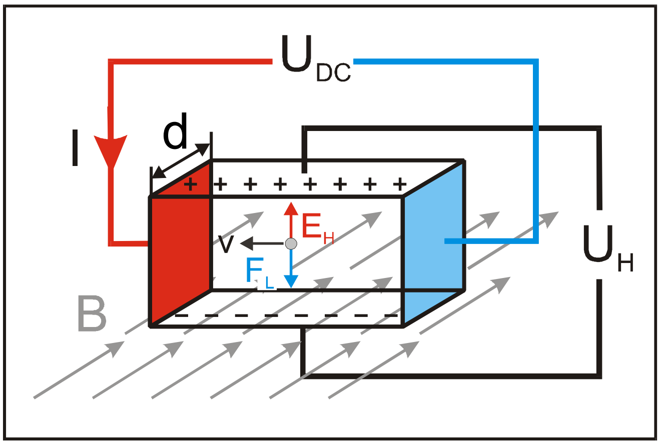
# Investigating the Hall effect in silver

## Objects of the experiment

- Validation of the proportionality of the Hall voltage and the magnetic flux density.
- Determining the polarity of the charge carriers.
- Calculating the Hall constant $R_H$ and the charge carrier concentration n.

Fig. 1: Hall Effect schematically: Inside a charge carrying metallic conductor which is located in the magnetic field B the Lorentz force $F_L$ is causing an electrical field $E_H$ resulting in a Hall voltage $U_H$. (I denotes the transverse current).



## Principles

If a current-carrying metallic conductor strip is located in a magnetic field B perpendicular to the direction of the current I, a transverse electrical field $E_H$ and a potential difference is produced (Hall effect).

The following equation holds for the Hall voltage $U_H$ (Fig. 1):

$$U_H = \frac{1}{n \cdot e} \frac{B \cdot I}{d} \tag{I}$$

B: magnetic flux density

I: current through the metallic conductor

d: thickness of the band-shaped conductor

n: concentration of charge carriers

$e = 1.602 \cdot 10^{-19}$ C: elementary charge

The Hall voltage $U_H$ is caused by the deflection of the moving charge carriers in the magnetic field due to the Lorentz force, whose direction may predicted by the right hand rule. The factor $\frac{1}{n \cdot e}$ is called Hall constant $R_H$:

$$R_H = \frac{1}{n \cdot e} \tag{II}$$

The sign of the Hall constant $R_H$ is determined by the polarity of the charge carriers.

The Hall constant depends on the material and the temperature. For metals $R_H$ is very small, however, for semiconductors $R_H$ becomes significantly large (compare experiments P7.2.1.3 and P7.2.1.4).

The polarity of the charge carriers can be determined from the direction of the Hall voltage. The concentration of the charge carriers n can be determined experimentally by measuring the Hall voltage $U_H$ as function of the magnetic field B for various currents I.

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Solid State Physics

Conduction phenomena
*Hall effect*

*LEYBOLD*
*Physics*
*Leaflets*

**P7.2.1.5**

# Determining the band gap of germanium

## Objects of the experiments

- Determining the voltage drop at an undoped Ge crystal as a function of the temperature when the current through the crystal is constant, and calculating the conductivity $\sigma$.
- Determining the band gap $E_g$ of germanium.

## Principles

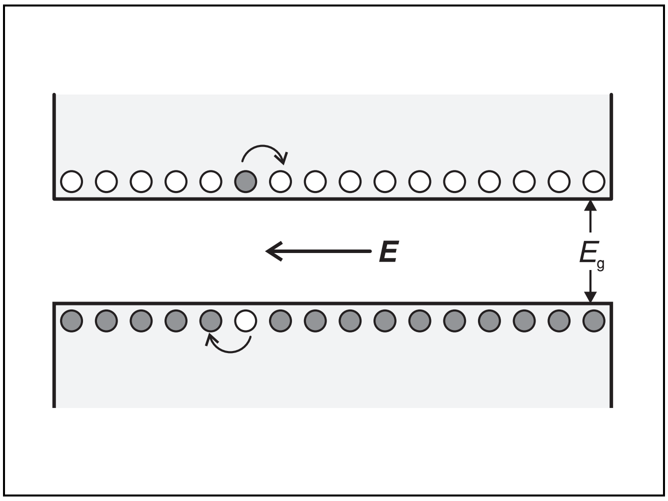
For the current density $j$ in a body under the influence of an electric field E the Ohm law states

$$j = \sigma \cdot E \qquad \text{(I).}$$

The proportionality factor $\sigma$ is called electric conductivity. Since this quantity strongly depends on the material, it is common to classify materials with regard to their conductivity. Semiconductors, for example, are solids that do not conduct electric currents at low temperatures, but show a measurable conductivity at higher temperatures. The reason for this temperature dependence is the specific band structure of the electronic energy levels of a semiconductor.

The valence band, i.e. the highest band that is completely or partially populated in the ground state, and the conduction band, i.e. the next unpopulated band, are separated by a band gap $E_g$ (Ge: $E_g \approx 0{,}7$ eV). The region between the two bands is not populated by electrons in an undoped, pure semiconductor and is called the “forbidden zone“. At higher temperatures, more and more electrons are thermally activated from the valence band into the conduction band. They leave “holes” in the valence band which move like positive charged particles, thus contributing to the current density $j$ as do the electrons (see Fig. 1).

Fig. 1 Simplified diagram of intrinsic conduction: a semiconductor with an electron in the conduction band and a hole in the valence band under the influence of an electric field $E$.



The conduction which is made possible by the excitation of electrons from the valence band into the conduction band is called intrinsic conduction. Since under conditions of thermal equilibrium the numbers of holes in the valence band and of electrons in the conduction band are equal, the current density in the case of intrinsic conduction can be written in the form

$$j_i = (-e) \cdot n_i \cdot v_n + e \cdot n_i \cdot v_p \qquad \text{(II)}$$

$e$: elementary charge,
$n_i$: concentration of electrons or holes respectively.

The average drift velocities $v_n$ and $v_p$ of the electrons and the holes are proportional to the field strength $E$. With

$$v_n = -\mu_n \cdot E \text{ and } v_p = \mu_p \cdot E \qquad \text{(III),}$$

where the mobilities $\mu_n$ and $\mu_p$ are chosen to be positive quantities,

$$j_i = e \cdot n_i \cdot (\mu_n + \mu_p) \cdot E \qquad \text{(IV)}$$

is obtained. Comparison with (I) leads to

$$\sigma_i = e \cdot n_i \cdot (\mu_n + \mu_p) \qquad \text{(V)}$$

for the conductivity. Apart from the elementary charge $e$, all quantities in (V) depend on the temperature $T$. The concentration of intrinsic conduction $n_i$ is

$$n_i \cdot = (N \cdot P)^{\frac{1}{2}} \cdot e^{-\frac{E_g}{2 \cdot kT}} \qquad \text{(VI),}$$

$k$: Boltzmann constant,
$E_g$: band gap of the semiconductor.

## Apparatus

| | |
|---|---|
| 1 Ge undoped on plug-in board . . . . . . | 586 851 |
| 1 base unit for Hall effect . . . . . . . . . . | 586 850 |
| 1 sensor CASSY . . . . . . . . . . . . . . | 524010 |
| 1 CASSY Lab . . . . . . . . . . . . . . . . | 524 200 |
| 1 current-controlled power supply, 15 V⎓, 3 A, for example . . . . . . . . . . . . . . . . | 521 50 |
| 1 power supply, 12 V⎓, 50 mA for example | 521 54 |
| 1 stand base, V-shape, 20 cm . . . . . . . | 300 02 |
| connection leads | |

$$N = 2 \cdot \left(\frac{2\pi \cdot m_n \cdot kT}{h^2}\right)^{\frac{3}{2}} \text{ and } P = 2 \cdot \left(\frac{2\pi \cdot m_p \cdot kT}{h^2}\right)^{\frac{3}{2}} \qquad \text{(VII)},$$

$h$: Planck constant,
$m_n$: effective electron mass,
$m_p$: effective hole mass,

are the effective state densities in the conduction band and in the valence band. The mobilities $\mu_n$ and $\mu_p$ also depend on the temperature. At low temperatures, the proportionality $\mu \propto T^{-\frac{3}{2}}$ holds roughly, as does the proportionality $\mu \propto T^{-\frac{3}{2}}$ at high temperatures.

Because of the predominance of the exponential function (see Eq. (VI)), conductivity is well approximated and represented by

$$\sigma_i = \sigma_0 \cdot e^{-\frac{E_g}{2 \cdot kT}} \qquad \text{(VIII)}$$

or

$$\ln \sigma_i = \ln \sigma_0 - \frac{E_g}{2 \cdot kT} \qquad \text{(IX)}.$$

With the object of confirming Eq. (VIII) and determining the band gap $E_g$, the conductivity of undoped germanium is determined as a function of the temperature $T$ in the experiment. At a constant current

$$I = j \cdot b \cdot c \qquad \text{(X)},$$

$b$: breadth of the crystal, $c$: thickness of the crystal,

the voltage drop

$$U = E \cdot a \qquad \text{(XI)},$$

$a$: length of the crystal,

is measured at an undoped Ge crystal.

Because of (I), (X), and (XI) the conductivity

$$\sigma = \frac{a}{b \cdot c} \cdot \frac{I}{U} \qquad \text{(XII)}.$$

## Setup

The experimental setup is illustrated in Fig. 2.

### Mounting and connecting the plug-in board:

*Notes:*

*The Ge crystal is extremely fragile:*
*Handle the plug-in board carefully and do not subject it to mechanical shocks or loads.*

*Due to its high specific resistance, the Ge crystal warms up even if only the cross-current is applied:*
*Do not exceed the maximum cross-current I = 4 mA.*
*Turn the control knob for the cross-current on the base unit for Hall effect to the left stop.*

- Insert the plug-in board with the Ge crystal into the DIN socket on the base unit for Hall effect until the pins engage in the holes.
- Turn the current limiter of the current-controlled power supply to the left stop, and connect the power supply to the input for the heating and electronics of the base unit for Hall effect.
- Switch the current-controlled power supply on, set the voltage limiter to 15 V and the current limiter to 3 A (for this, short-circuit the output of the power supply temporarily).
- Turn the control knob for the cross-current on the base unit for Hall effect to the left stop, and supply the current source by connecting the other power supply. Switch this power supply on and set its output voltage to 12 V.

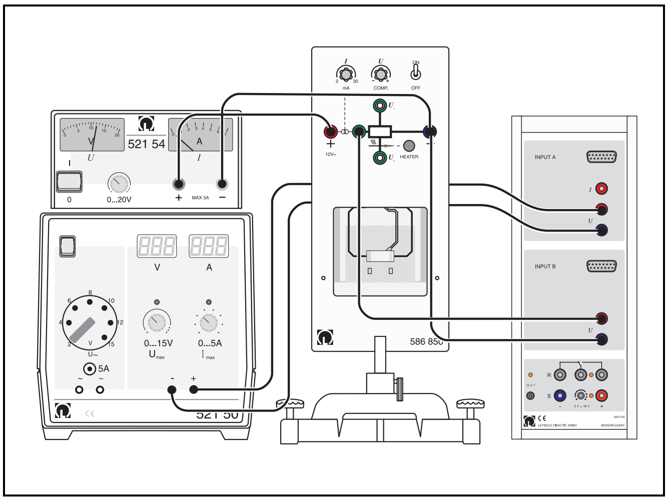


Fig. 2 Experimental setup for the determination of the band gap of germanium

### Preparation for data logging:

- Connect the output for the temperature measurement to input A and the output for the voltage drop to input B of sensor CASSY.
- Connect sensor CASSY to the serial interface of the PC (usually COM1 or COM2) with the nine-pole V24 cable.
- If necessary, install the “CASSY Lab” program under Windows 95/98/NT and choose the desired language.
- Get the “CASSY Lab” program started and check whether connection of the CASSY sensor is correct.

# Solid-state Physics

Conduction phenomena
*Hall Effect*

***Physics Leaflets***

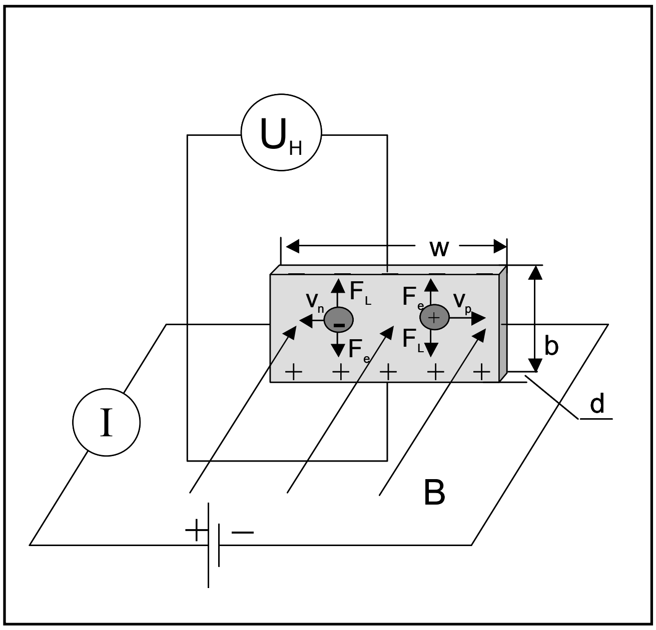
**P7.2.1.4**

# Determining the density and mobility of charge carriers in p-germanium

## Objects of the experiment

- Measuring of the Hall voltage as function of the current at a constant magnetic field: determination of the density and mobility of charge carriers.
- Measuring of the Hall voltage for as function of the magnetic field at a constant current:: determination of the Hall coefficient.
- Measuring of the Hall voltage as function of temperature: investigation of the transition from extrinsic to intrinsic conductivity.

Fig. 1: Hall effect in a rectangular sample of thickness d, height b and length w: At equilibrium conditions the Lorentz force $F_L$ acting on the moving charge carriers is balanced by the electrical force $F_e$ which is due to the electric field of the Hall effect.



## Principles

The Hall effect is an important experimental method of investigation to determine the microscopic parameters of the charge transport in metals or doped semiconductors.

To investigate the Hall effect in this experiment a rectangular strip of p-doped germanium is placed in a uniform magnetic field B according Fig. 1. If a current I flows through the rectangular shaped sample an electrical voltage (Hall voltage) is set up perpendicular to the magnetic field B and the current I due to the Hall effect:

$$U_H = R_H \cdot \frac{I \cdot B}{d} \qquad \text{(I)}$$

$R_H$ is the Hall coefficient which depends on the material and the temperature. At equilibrium conditions (Fig. 1) for weak magnetic fields the Hall coefficient $R_H$ can be expressed as function of the charge density (carrier concentration) and the mobility of electrons and holes:

$$R_H = \frac{1}{e_0} \cdot \frac{p \cdot \mu_p^2 - n \cdot \mu_n^2}{(p \cdot \mu_p + n \cdot \mu_n)^2} \qquad \text{(II)}$$

$e_0 = 1.602 \cdot 10^{-19}$ As (elementary charge)
$p = p_E + p_S$ (total density of holes)
$p_E$: density of holes (intrinsic conduction)
$p_S$: density of holes (hole conduction due to p-doping)
$n = n_E$ :density of electrons (intrinsic conduction)
$\mu_p$:mobility of holes
$\mu_n$ mobility of electrons

From equation (II) follows: The polarity of predominant charge carriers can be determined from the Hall coefficient $R_H$ if the directions of the current I and magnetic field B are known. The thinner the conducting strip the higher the Hall voltage.

The doping of group III elements like e.g. B, Al, In or Ga into the crystal lattice of germanium creates positive charged holes in the valence band (Fig. 2).

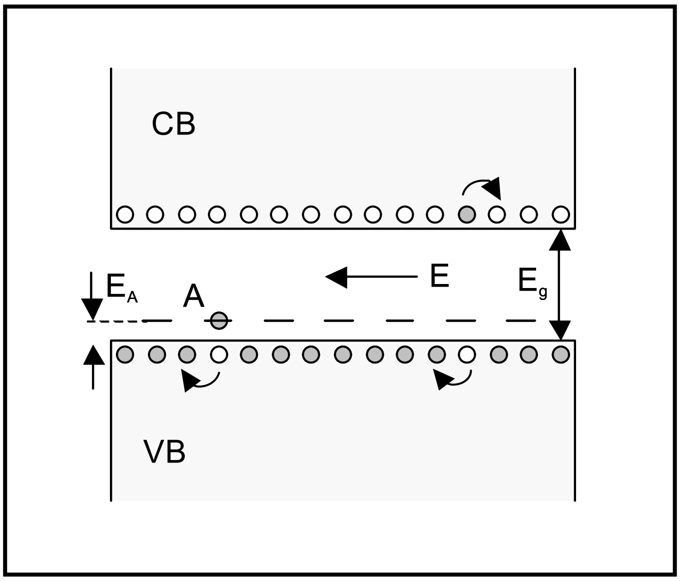


Fig. 2: Simplified diagram of extrinsic (left) and intrinsic conduction (right) under influence of an electric field E: Incorporating of dopants (acceptors A) into the crystal lattice creates positive charge carriers called holes in the valence band (VB). With increasing temperature the thermal energy of valence electrons increases allowing them to breach the energy gap $E_g$ into the conduction band (CB) leaving a vacancy called hole in the VB.

Their activation energy $E_A$ of about 0.01 eV is significantly smaller than the activation energy $E_g$ (band gap) to generate electrons and holes by thermal activation (intrinsic charge carriers). At room temperatures in p-doped germanium the density of holes $p_S$ can predominate the density of intrinsic charge carriers ($p_E$ and $n_E$). In this case where the charge transport is predominately due to holes from the dopants ($n = n_E = p_E \approx 0$). The density of $p_S$ can be determined by measuring the Hall voltage $U_H$ as function of the current I. With equation (I) and (II) follows:

$$p_S = \frac{B}{e_0 \cdot d} \cdot \frac{I}{U_H} \qquad \text{(III)}$$

The mobility is a measure of the interaction between the charge carriers and the crystal lattice. The mobility is defined as (in case p-doped germanium it is the mobility $\mu_P$ of the holes created by the dopants, i.e. acceptors):

$$\mu_P = \frac{v_p}{E} \qquad \text{(IV)}$$

$v_p$: drift velocity

E: electric field due to the voltage drop

The electric filed E can be determined by the voltage drop U and the length w of the p-doped germanium strip:

$$E = \frac{U}{w} \qquad \text{(V)}$$

The drift velocity $v_p$ can be determined from the equilibrium condition, where the Lorentz force compensates the electrical force which is due to the Hall field (Fig. 1)

$$e_0 \cdot v_d \cdot B = e_0 \cdot E_H \qquad \text{(VI)}$$

which can be expressed using the relation $E_H = b \cdot U_H$ as

$$v_d = \frac{U_H}{b \cdot B} \qquad \text{(VII)}$$

Substituting equation (V) and (VII) in equation (IV) the mobility $\mu_p$ of holes can be estimated at room temperatures as follows:

$$\mu_p = \frac{U_H \cdot w}{b \cdot B \cdot U} \qquad \text{(VIII)}$$

The current I in a semiconductor crystal is made up of both hole currents and electron currents (Fig. 1):

$$I = b \cdot d\,(n_p \cdot \mu_p + n_n \cdot \mu_n) \qquad \text{(IX)}$$

The carrier density depends on the dopant concentration and the temperature. Three different regions can be distinguished for p-doped germanium: At very low temperatures the excitation from electrons of the valence band into the acceptor levels is the only source of charge carriers. The density of holes $p_S$ increases with temperature. It follows a region where the density $p_S$ is independent of temperature as all acceptor levels are occupied (extrinsic conductivity). In this regime the charge transport due to intrinsic charge carriers can be neglected. A further increase in temperature leads to a direct thermal excitation of electrons from the valence band into the conduction band. The charge transport increases due to intrinsic conductivity and finally predominates (Fig. 2). These transition from pure extrinsic conduction to a predominately intrinsic conduction can be observed by measuring the Hall voltage $U_H$ as function of the temperature.

To describe the Hall voltage as function of temperature $U_H$ based on a simple theory equation (I) and (II) have to be extended in the following way:

It is assumed that the mobility of electrons and holes are different. Introducing the ratio of the mobility

$$k = \frac{\mu_n}{\mu_p} \qquad \text{(X)}$$

equation (II) can be rewritten as follows:

$$R_H = \frac{1}{e_0} \cdot \frac{p - n \cdot k^2}{(p + n \cdot k)^2} \qquad \text{(XI)}$$

For undoped semiconductors the temperature dependency of the charge carriers can be assumed as

$$n = n_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}}$$

$$p = p_0 \cdot e^{-\frac{E_g}{2 \cdot k_B \cdot T}} \qquad \text{(XII)}$$

$k_B = 1.36\ 10^{-23}$ J/K: Boltzmann constant

The product of the densities p and n is temperature dependent:

$$n \cdot p = n_E \cdot (p_E + p_S) = \eta^2 \qquad \text{(XIII)}$$

where the effective state density $\eta$ is approximated as

$$\eta^2 = N_0 \cdot e^{-\frac{E_g}{k_B \cdot T}} \qquad \text{(XIV)}$$

In the extrinsic conductivity regime the density $p_S$ of holes can be determined according equation (III). For the intrinsic charge carriers $p_E = n_E$ which leads to a quadratic equation for $p_E$ with the solution:

$$p_E = -\frac{P_S}{2} + \sqrt{\frac{P_S^2}{4} + \eta^2} \qquad \text{(XV)}$$

With equations (XI) and (XV) together with the relations $p = p_E + p_S$ and $n = n_E$ the temperature dependency of Hall voltage $U_H$ can be simulated. Using for $E_g$ = 0.7 eV the result of experiment P7.2.1.5 as estimate value for the simulation only two unknown parameters $N_0$ and k are left.