UNIVERSITY OF WATERLOO

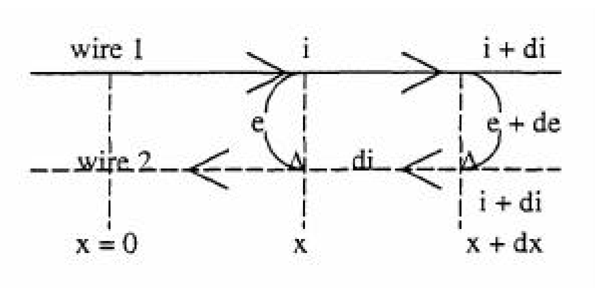
Physics 360/371 – Experiment 7

WAVES AND PULSES IN CABLES

References: Feynman Lectures on Physics. Vol. II, 24 - 1. Addison-Wesley 1964.
Electronics, T.B. Brown, Ch. 13, pp. 422-424.

Transmission cables usually take the form of coaxial conductors separated by a dielectric. The outer conductor is usually grounded. The cable is characterized by certain values of inductance and capacitance per unit length. Two lines, one of them dashed, will represent a cable.

We will limit ourselves to lossless cables in the analysis, that is, cables in which the resistance of the conductors and the conductance of the dielectric are both zero. In general, the current and voltage will be functions of x and t.



We use the simple relationships between voltage (e), current (I), capacitance per unit length (c) and inductance per unit length (L). The first equation relates the voltage change in the length dx to the current through it:

That is,

$$\frac{\partial e}{\partial x}dx = -L\frac{\partial i}{\partial t}dx$$

or,

$$\frac{\partial e}{\partial x} = -L\frac{\partial i}{\partial t} \qquad \text{(I)}$$

The second equation relates the current change in a length dx to the capacitance between conductors, that is

$$\frac{\partial i}{\partial x}dx = -C\frac{\partial e}{\partial t}dx\,,$$

or,

$$\frac{\partial i}{\partial x} = -C\frac{\partial e}{\partial t} \qquad \text{(II)}$$

From these equations, we can arrive at the wave equation, i.e.

$$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2}, \qquad \text{(IIIA)}$$

and

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \qquad \text{(IIIB)}$$

We can assume solutions for an infinite cable (no reflections from ends) of the type:

$$e = e_1 \exp[j(wt - kx)] + e_2 \exp[j(wt + kx)] \qquad \text{(IVA)}$$
$$i = i_1 \exp[j(wt - kx + f_1)] + i_2 \exp[j(wt + kx + f_2)] \qquad \text{(IVB)}$$

Putting these equations into IIIA and IIIB, we can easily show that

$$(k/w)^2 = LC \qquad \text{(V)}$$

But, of course, the speed of the wave is given by w/k. Thus, the speed of the wave is related to the inductance and capacitance per unit length of cable. If we put (IVA) and (IVB) into (I) and (II), we can also show that $f_1 = f_2 = 0$ and that for all x and t we have $e/i = \sqrt{L/C}$. Since e/i is the ratio of voltage to current, the quantity $\sqrt{L/C}$ is called the impedance of the cable and is given the symbol R'. For a cable with vacuum as a dielectric, the signal speed is the speed of light in vacuum. For typical cables, the speed is about 75% of that value.

Since the ratio e/i is R' for all x and t in an infinite cable, it is easy to see that if one sends a signal into the end of a semi-infinite cable, the cable acts like a pure resistance of value R'. Also, if one has a finite cable and terminates it with a resistor R', it acts as a semi-infinite cable. No signal will be reflected back. The condition e/i = R' is satisfied by the end resistor, and all the power will be absorbed by it. This sort of termination is used to eliminate reflected signals.

Two other cases are easy to analyze. If the far end ($x = \ell$) is open circuited, the current there must be zero (a node). Thus, we must have a reflected wave 180° out of phase with the incident wave. We can represent the total current in the cable by:

$$i = i_0[\exp j\{wt - k(x - \ell)\} - \exp j\{wt + k(x - \ell)\}] \qquad \text{(VI)}$$

Also, for such a termination, the voltage amplitude would be a maximum (anti-node) at $x = \ell$. The voltage could then be written:

$$e = e_0[\exp j\{wt - k(x - \ell)\} + \exp j\{wt + k(x - \ell)\}] \qquad \text{(VII)}$$

This cable presents an impedance to the generator of e(0,t) / i(0,t), which we can show to be

$$Z = -jR' \cot k\ell \qquad \text{(VIII)}$$

Similarly, in ease the far end is shorted, we have a current anti-node and a voltage node at x = l . Equations similar to (VI) and (VII) can be written and we get an impedance at x = 0 of

$$Z = +jR' \tan k\ell \qquad \text{(IX)}$$

If pulse signals are sent down the cable, we will get different sorts of signals reflected, depending on the termination. If it is R', nothing comes back. If it is an open circuit, we get erect voltage pulses (same as those sent) and inverted current pulses. If it is shorted, the opposite occurs. It should be noted that the analyses assumed no losses. In practice, the signals are attenuated exponentially as they travel along the cable. The student should consider the effect of this on reflected waves and pulses.

Experiment:

I. Calculation of L and C

Measure the diameters of the inner conductor and the insulation and calculate L and C. Use methods from any E & M text for this. Note that the dielectric constant is not known at this point.

II. RF Input

(a) Connect the 60 meter cable to the function generator. An ordinary oscilloscope is used to display both the input and reflected voltage. The current signal is obtained by means of a current "probe", clipped over the conductor. It is essentially a transformer, using as a primary the conductor in which the current is to be measured. Set the frequency to about 3 MHz. Set the buffer source impedance to the 75 ohm location and its output mode to CW.

(b) Vary the frequency (1 – 10 MHz) and determine the resonant frequencies at which Z = Z_{min} defined by v_{min}, i_{max} where F = 0°. About two successive resonances, obtain adequate numbers of voltage and current readings as a function of frequency so that you can properly plot the impedance. Do this for the end of the cable open and closed (shorted).

Plot impedance as a function of frequency for these two cases.

(c) From the measurements of frequencies for $Z = Z_{min}$ and employing $\Delta f = f_{n+1} - f_n$, determine the speed of propagation and its associated error of the wave along the cable. Derive the equation used by considering the case for which Z = 0. From the determined speed, and the calculated values of L and C, determine the dielectric constant of the insulator. What is the name of your dielectric?

(d) Observe and record the phase relationship near resonance ($Z = Z_{min}$) between the current and the voltage waveforms for the end of the cable open, shorted and terminated by a resistor. Repeat and compare your results for the case of $Z = Z_{max}$ defined by v_{max}, i_{min} where F = 0°. What kind of analogous LCR circuits give rise to such phase trends for the $Z = Z_{min}$ and $Z = Z_{max}$ resonant conditions?

(e) Derive and calculate the resonant frequency predictions deduced from standing waves in the cable produced with a voltage node at the generator end and (i) an anti-node for the open and (ii) a node for the shorted end. Compare these predicted frequencies with those measured.

III. Pulse Input

(a) Connect the 60 meter cable to the Hewlett Packard Pulse Generator with the buffer set to the pulse mode. Connect the current and voltage probes to the oscilloscope as in part II. Adjust the width control to obtain narrow pulses of approximately 22 ns in duration. Work at a frequency of a few hundred kHz.

(b) Study the reflection of pulses from the open end of the cable, with the end shorted and with a matched load on the end. Take a photograph of the three cases. When the matched load resistance has been determined by experiment (use an ohmmeter), relate it to your value calculated using L and C, and the dielectric constant obtained in II(e). From the scope display or the photographs and the calibrated time base determine the speed of propagation of the pulse and its uncertainty and compare it with that of II(c).

(c) Repeat (b) with an 18 meter then 9 meter cable and comment on the changes. (No photographs required).

(d) Observe and account for the pulse trains you see when the buffer impedance is set greater than the characteristic impedance of the cable and less.

(e) Calculate L and C for the cable using the relations between R' and L, C and between v and L, C.

Equipment List:

- H.P. pulse/function generator
- CW & Pulse Mode Buffer
- 1 variable resistance terminal box 0 – 110O
- 1 current probe tap
- BNC connectors, various
- 1 BNC shorting termination
- 1 60m cable
- 3 9m cables
- Hameg Oscilloscope Model 205
- Voltage probe
- Oscilloscope camera