Physics 360/460 Intermediate Laboratory
Experiment #25
Measurements of Acoustic Systems with Maximum-Length Sequences

Abstract

A maximum-length sequence (MLS) is a computer-generated sequence of 1s and 0s that acts like random noise, but is deterministic and periodic. Excitation and subsequent cross-correlation by an MLS is an efficient time-domain method to identify a system. The resulting periodic impulse response can be transformed to the frequency domain where it becomes the frequency response of the system. Such a system (CLIO) is used to measure an electronic filter, make quasi-anechoic measurements of a loudspeaker in a normal room, and finally to evaluate the reverberation decay of a room in octave bands.

Background

In order to measure the gain and phase of a linear system as a function of frequency, the so-called frequency response, it is necessary to excite the system with a signal that covers completely the frequency band of interest. Signals at both the input and output must be captured and analyzed to obtain this response. One such suitable signal is the Dirac δ-function, which has a Fourier transform that is a constant at all frequencies, representing a perfect response. This particular input signal has a known spectrum, so it is only necessary to measure the output of the system to determine the system response.

If a delta-function input signal $\delta(t)$ is applied to a linear system, the resulting output is called the impulse response h(t). Now, the input x(t) can be thought of as a series of weighted δ-functions, such that:

$$x(t) = \int x(\tau)\, \delta(t-\tau)\, d\tau.$$

The integrand only gives a contribution when the argument of the δ-function goes to zero, when τ=t. The δ-function has unit area, and when integrated gives a contribution of unity, multiplying the factor $x(\tau)$ which thus gives x(t). This is the simplest of convolutions; convolution by a δ-function does not change a function.

If a signal x(t) is applied to the system, it can be written as an integral of weighted δ-function contributions as above. The output of each δ-function results in the response $h(t-\tau)$, but now weighted by the function $x(\tau)$. Hence the output y(t) is the convolution of h(t) with x(t). The result is (the symbol ** stands for convolution):

$$y(t) = h(t) ** x(t) = \int h(t-\tau)\, x(\tau)\, d\tau, \text{ or due to symmetry, } = \int x(t-\tau)\, h(\tau)\, d\tau.$$

It is easily shown that the Fourier transform of a convolution is the product of the Fourier transform of the two functions in the convolution, so that:

$$Y(\omega) = H(\omega)\,X(\omega),$$

in which each capitalized term is the Fourier transform of the corresponding time-domain function. Now, $Y(\omega)$ and $X(\omega)$ are respectively the frequency spectra of the input and output signals of the system under consideration, and thus $H(\omega)=Y(\omega)/X(\omega)$ is the desired frequency response of the system, being a complex number that is usually represented as magnitude response and phase angle. But $H(\omega)$ is the Fourier transform of h(t), the impulse response of the system. *Thus we have shown that the impulse response and the frequency response are related by the Fourier transform*. We call h(t) the time-domain response of the system, and $H(\omega)$ its associated frequency-domain response. They are interchangeable under Fourier transformation. Either is a complete description of the system.

In the CLIO measurement system, instead of a δ-function, a noise-like sequence is generated from the binary output of a shift register containing 14 bits. If outputs 2, 12, 13 and 14 (that is one possible configuration) are fed to an exclusive-OR tree and fed back to the input, the register goes through all possible states in quite a random way, except the all-zero state which would perpetuate just itself. Thus such an MLS has a length of $2^{14}-1=16383$ samples. At a sampling rate of 51.2 kHz, it would repeat about 3 times a second. This is easily heard when such a signal is fed to a loudspeaker, as in the second experiment of this laboratory. The 1's and 0's produced by the shift register are mapped to signals of −V and +V, in which case the exclusive-OR operations are replaced by multiplication.

In this experiment, the time-domain response of the system under test is measured by the aforementioned MLS technique. Instead of a δ-function, the input signal is the MLS-generated sequence of +V's and −V's. However, *the cyclic auto-correlation function of this sequence is essentially a δ-function*. It is easily shown that when the system output is cyclically *cross-correlated* with the original sequence of +1's and −1's, the result is the cyclic or periodic impulse response h(t) of the system, just as though an actual impulse had been used. There is a surprisingly efficient way to do the cross-correlation since it requires only addition and subtraction. Note that the MLS excitation sequence has much nicer signal properties than a δ-function, which is usually represented by a single sample of quite high amplitude. This can cause overload or other unwanted behaviour, but is completely avoided by the MLS signal.

In order to make the impulse response meaningful, it should decay adequately in one period (16383 samples). This is easily arranged for most typical measurements. The computer can readily Fourier transform the impulse response h(t) to give the frequency response $H(\omega)$.

In this experiment, the signals are all represented by discrete-time sequences. All the foregoing theory is valid for discrete time, with the Discrete Fourier Transform (DFT) taking the place of the continuous-time transform. This transform is easily calculated by computers, since very fast algorithms are available which exploit the features of signals that can be represented by 2^M data points, where M is an integer. For the CLIO system, M=13, which results in 8192 data points. This represents a total analysis time of 160 ms at a sampling frequency of 51.2 kHz. When transformed the DFT gives 4096 complex frequency points, separated in frequency by 6.25 Hz.

The frequency domain is usually displayed in magnitude (often in dB) and phase angle (°). The dB or decibel is defined by:

$$dB = 20 \log (\text{magnitude}/[\text{reference magnitude}]) = 10 \log (\text{power}/[\text{ref. Power}]).$$

The reference magnitude is often chosen as the measurement when certain initial conditions are applied to the system, for example, in a loop-back measurement in which we would consider the gain to be unity.

The resolution of a measuring system is determined by the length of time it is observed. An observation time τ results in a frequency resolution $1/\tau$. This means that if we truncate an impulse response to say, 3 ms, the resolution will be 333 Hz, even if we use a very large number of points in the DFT. The data will be smooth, but the resolution will be poor. In the CLIO system the DFT is always 160 ms long, corresponding to a frequency separation between points of 6.25 Hz, but the resolution will depend on the actual length of the chosen data portion.

Preliminary Experiment: Normalization of the gain and time delays.

With the cables attached to the A-channel input and output (see first few pages of the manual), connect them together for a loop-back measurement. The MLS Analyze mode should be used. Signal levels should be around 1 volt. The periodic impulse response will show some time delay. To go to the time domain click the X in the upper right corner of the frequency domain display. This is due to the anti-aliasing filter in the analogue-to-digital converter, and to other clocked processes. Record this minimum delay for your later experiments. The frequency response (click FREQ) gives the response in dB that represents unity gain for the analyzer settings. Note it as well. Students should find that the loop-back experiment gives a very sharp h(t) and a flat $H(\omega)$.

Experiment 1: Time and Frequency responses of a filter.

Simple resistor-capacitor networks can provide low-pass (LP) or high-pass (HP) filtering operations. Make sure you understand (from P252/253 or P352 courses) the expected time- and frequency-domain responses from such devices. For simple RC filters, the angular frequency ω of the breakpoint is given by:

$$\omega RC=1.$$

Measure each of the networks you are given and print out the impulse and frequency responses over appropriate ranges. Calculate the theoretical time constant of each network and compare with the value measured from the graph. Calculate the measured frequency breakpoint of each network (where the two asymptotic straight lines meet) and compare with theory. Note that the gain of the low-pass filter is not close to unity. Explain this behaviour, which is due to the 64k-ohm input resistance of the CLIO input. This resistance represents a significant load relative to the resistors in the filter. It must also be taken into account to calculate the time constants and break-point frequencies for both the LP and HP filters.

Experiment 2: Frequency response of a loudspeaker.

Connect the CLIO input and output to the external amplifier box. This unit contains a power amplifier to drive a loudspeaker, and also supplies power to the active microphone that is plugged into it. The mic has a rubber guard that should be removed when in use. Connect the loudspeaker to the amplifier output binding posts, and place the microphone about ½ m directly in front of it, on the tweeter axis. Remove all possible obstructions to maximize the reflection-free time. Do an analysis and view the impulse response. The extra acoustic delay should be clearly visible, with a sharp initial response followed by reflections from the room. Print the response.

The portion of h(t) chosen for frequency analysis can be set by moving the cursor to the desired spot and using the START and STOP buttons. Print out frequency responses for all the time data (160 ms), and for truncated "quasi-anechoic" impulse responses using points 1 ms and 5 ms past the start of the data. Compare the frequency responses and comment. By looking at the very first portion of h(t), make an estimate of the speed of sound. You will need to correct for the minimum delay measured in the preliminary experiment. Comment on why it is not possible in a normal room to quasi-anechoically measure the lowest frequencies.

Repeat the measurement with the loudspeaker and the mic placed near a wall so that a strong reflection occurs, say 2 ms after the main impulse response, using 5 ms of time data. Explain the undulations in the frequency response, and their separation in frequency.

Experiment 3: Reverberation of a Room

When sound produced in a room is suddenly turned off, it usually decays exponentially, so the plot of dB versus time is a straight line. The reverberation time RT60 is defined as the time for the sound to decay by 60dB, to one millionth of its power. The noise often prevents decay by 60dB, so the slope of the linear portion is extended to make the determination. The decay in each frequency range can be calculated by filtering h(t), squaring it, and time-reverse integrating the result. This "Schroeder plot" is a very useful derivation from the impulse response, and the procedure is implemented by the CLIO system.

Place the loudspeaker quite far from the microphone to simulate a source and more distant listener. Set up the CLIO system to measure the reverberation decay (Acoustics: RT60). Click all the frequency ranges from 63 Hz to 8 kHz. In this mode the system measures the impulse response for various filtered excitations at several different sampling rates, and uses them to calculate the data. The calculation of the reverberation time is carried out between the black horizontal lines (you can select the levels) on the integrated Schroeder plot that the CLIO system provides. You will have to employ a hand analysis of the data since the noise floor generally prevents the decay from being sufficient. Print the decay curves for 125, 500, 2000, and 8000 Hz (expand the horizontal axis if necessary) to do the calculation.

Plot the reverberation time above as a function of frequency. Repeat the experiment with the door to the room left open, or placing absorbers in the room if available. Explain the results. Comment on the variation of the reverb time at both the lower frequencies and higher frequencies. The room used for the experiment has drywall panels which have quite a lot of low -frequency absorption.
