UNIVERSITY OF WATERLOO
Physics 360/460 Experiment # 17

The Analogue Computer

The Operational Amplifier (op amp) is the essential element of the analogue computer. By using a combination of op amps, along with electronic multipliers and dividers, it is even possible to solve differential equations. The circuit's output voltage, V_o, corresponds to the solution to the circuit's operation on the input voltage(s).

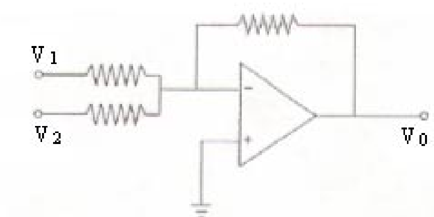
A rack containing a number of op amp's along with provisions for wiring is provided along with an assortment of plug-in resistors and capacitors that make up the computer's circuit. In addition to the potentiometers that will be useful in various circuits, at the top of the rack are provisions for electronic switches that can be driven, for example, by a square wave from the Function Generator near the bottom of the rack. Moreover, you can also use the square wave output to trigger the digital storage oscilloscope which may provide a better, jitter free, display of the circuit output. A second Function Generator that has a variable phase output is on the bench. *Keep in mind: the scope may be ac or dc coupled to provide an optimum display, avoid possible ground loops and make absolutely certain that you keep the input voltages in the range of a few volts (or less) so as not to overload the op amps (output at rail voltage).*

After setting up the circuit and observing its output voltage via the oscilloscope, you can use the XY pen recorder to plot the observed waveform. This is done by using the "hold" and "plot" buttons near the top of the oscilloscope.

There are multiplying circuits at the rack's top that are useful in setting up more complicated problems such as nonlinear differential equations; see appendix.

In the circuits below start by using R = 100 kΩ, and C = 0.1 μF and frequencies << 100Hz unless otherwise noted. Circuit symbols have their usual meaning.

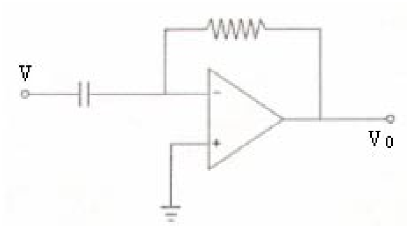
1) Summing



$V_o = -(V_1 + V_2)$

Apply a combination of sine, square and triangular waves to the (V_1 and V_2) inputs and observe the output. (i.e. by using the variable phase ability of the HP oscillator you can vary the input phase between two sine (or square or sine and square) waves. For all cases demonstrate that $V_o = -(V_1+V_2)$

2) Differentiation



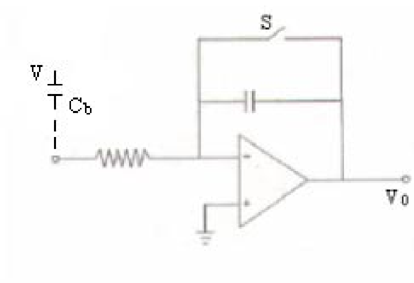
$$V_o = -RC\frac{dV_i}{dt}$$

Apply sine, square and triangular waves to the input, and observe and plot the output. Compare the output and input by means of the dual beam oscilloscope and verify that the circuit is differentiating.
You may then also want to add +/- DC offsets along with voltage ramps to the input. Can the oscilloscope be DC coupled?

3) Integration

$$V_0 = \frac{-1}{RC}\int dV_i dt$$

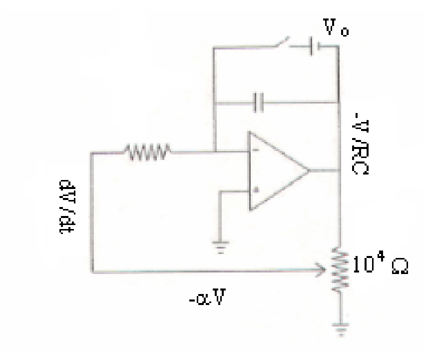
note: capacitor C_b is not integral to the circuit but simply there to block possibly undesirable dc voltages



Begin by applying a square wave (output is about 1/10 of the input voltage; switch, S, is not needed). Try sine and triangle waves as well. Next, remove C_b and apply a DC voltage (+3V) to the input and observe the voltage 'ramp' at the output. To do so you need to 'reset' voltages with the switch, or else the output simply reaches the fixed rail voltage (use the oscillator's dc offset to generate a positive 6V square wave to trigger switch S; watch the polarity). Calculate the ramp's rate of rise from the circuit values and compare to your results. Vary input voltages and comment; (i.e. explain 'polarity' of the output). Can the oscilloscope be DC coupled?

4) Exponential Functions

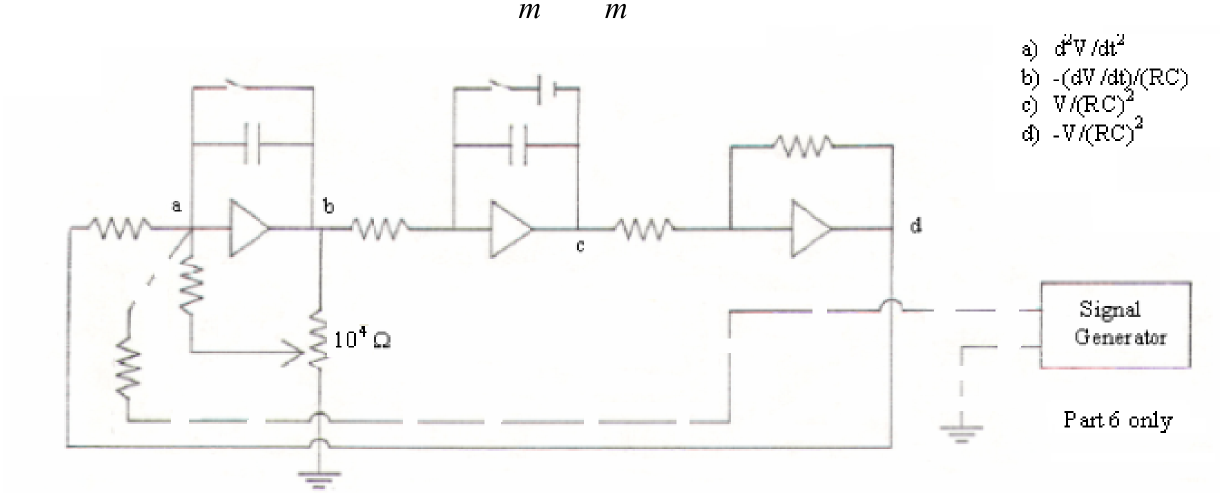
$$\frac{dV}{dt} + \alpha V = 0$$



This equation, realized by the adjacent circuit, has solution $V = V_0 e^{-\alpha t}$. Measure the voltages and compare with the solution. Explain how and where 'αV' and 'dV/dt' are implemented in the circuit; note, $\alpha = \beta/RC$ where β is the potentiometer ratio that varies from 0 to 1

5) Damped Harmonic Motion

$$\ddot{V} + \frac{b}{m}\dot{V} + \frac{k}{m}V = 0$$



The circuit above corresponds to the damped harmonic motion equation. All op amps use a “-“ input with “+” grounded. Explain and verify the voltages indicated at the various points and correlate them to the differential equation. Note: $(b/m) = \beta/RC$; $(k/m) = 1/(RC)^2$. Identify the function of each of the circuit components. Measure the frequency at zero damping and compare with $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Measure the logarithmic decrement produced with moderate damping. The amplitude’s decay should correspond to $A = A_o e^{-\lambda t}$ where $\lambda = b/2m$. Verify this and discuss.

6) Forced Damped Harmonic Motion: $\ddot{V} + \frac{b}{m}\dot{V} + \frac{k}{m}V = \frac{F_0}{m}\cos(\omega t)$

Measure the amplitude in the steady state as a function of ω. From this obtain the Q value (Quality factor) and compare with $\omega_0 \frac{m}{b}$ where $\omega_0 = \sqrt{\frac{k}{m}}$.

(NOTE: Circuit of part 5 must be modified as shown.)

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