E4 Brown, Electronics, pp 216-222



which is described in Chapter 8. In this phototube the current is amplified many times by ionization of gas atoms. Another method for amplification of phorotube currents is the electron-multiplier tube described in Sec. 20, in which amplification is accomplished by secondary-electron emission.

8. Thermionic Emission. Thermionic emission of electrons from a hot cathode may be accurately described as an *evaporation* process. It should not be confused with boiling; the correct analog is instead the escape of water molecules from the surface of water liquid which is at a temperature far below the boiling point. In this analog, water molecules are moving about within the liquid with speeds for which the root mean square value, $\bar{u}$, is determined by the temperature of the liquid, in accord with the kinetic-theory equation* for the average kinetic energy of a molecule. If $T$ is the absolute temperature of the liquid, $m$ the mass of each molecule, and $k$ the Boltzmann constant (the gas constant per molecule), then

$$\tfrac{1}{2}m\bar{u}^2 = \tfrac{3}{2}kT \tag{6}$$

If all the molecules had the same speed, $\bar{u}$, none would be able to escape through the surface of the liquid until the boiling point was reached, and the fact that evaporation does occur proves the true motion to be quite different. The speed for individual molecules may range from very high to very low values, although those having speeds near $\bar{u}$ outnumber greatly those whose speeds depart far from this value. Thus there are always a few molecules moving toward the surface with enough speed to escape into the space above the liquid, and the number of these increases rapidly as $T$ increases.

Richardson explained thermionic emission of electrons in like manner. As he pictured it, the *conduction* electrons in a metal (one, or sometimes two, for each atom) move freely about among the atoms with an average kinetic energy determined by the temperature of the metal, in accord with Eq. 6. The electron current in a thermionic tube then is maintained by the very small fraction of the electrons which are able to escape from the metal, against the forces binding them to it, because they possess in the metal kinetic energies far above the average and are moving toward the surface.

9. Richardson's Equation. From these assumptions Richardson derived the following equation for the number of electrons emitted per unit area of a metal surface, per unit time.

$$n = a\sqrt{T}\epsilon^{-w'/kT} \tag{7}$$

* For this and other references to the kinetic theory of matter, see Chapters XI and XII in T. B. Brown, *Foundations of Modern Physics*.

Since the saturation current is obtained when these electrons are carried to the anode as fast as they are emitted, so that none return to the cathode, the saturation current density (current per unit area of cathode) corresponding to Eq. 7 is

$$J = ne = ae\sqrt{T}\epsilon^{-w'/kT} \tag{8}$$

In Eqs. 7 and 8, $a$ is an integration factor, $\epsilon$ is the Naperian constant, 2.72, and the quantity $w'$ is the *work-function* of the surface and represents the work done in removing one electron from it. In the evaporation analog $w'$ corresponds to the heat of vaporization *per molecule*.

We now know that certain of Richardson's assumptions are wrong; this will be explained in Sec. 18. When the correct assumptions are made the equation for the saturation current density becomes

$$J_0 = AT^2\epsilon^{-w_0/kT} \tag{9}$$

Equation 9 may be called Richardson's corrected equation, or Dushman's equation. In it $w_0$ is the true work-function for the surface, and $A$ is another constant determined by the nature of the surface. Theory indicates that $A$ should be 120 amp cm$^{-2}$, but experimental values do not agree with this figure, and are different for each type of surface. It is possible to account for these differences in several ways, although the experiments do not show which is the correct explanation.

Experiments to verify this equation encounter several difficulties. First of all, the thermionic current does not reach a constant value as the anode potential is raised but continues to rise slowly, even when the potential is so large that all electrons are drawn away from the cathode to the anode as fast as they are emitted. Evidently the rate of emission is being increased by the anode potential, or rather, by the electric field produced at the cathode by that potential.

**10. The Schottky Effect.** Schottky showed that the effect of an external field, $E$, is to reduce the work-function of the surface by an amount equal to $\sqrt{Ee^3}$,* so that the work-function under these conditions is

$$w = w_0 - \sqrt{Ee^3} \tag{10}$$

When there is an external field, $w$ must replace $w_0$ in Eq. 9, and the saturation current density is $J$ instead of $J_0$:

$$J = AT^2\epsilon^{-w/kT}$$

$$= AT^2\epsilon^{-w_0/kT}\epsilon^{\sqrt{Ee^3}/kT} \tag{11}$$

* See Appendix 4 for this proof.

or, from Eq. 9,

$$J = J_{0\epsilon}\sqrt{E_c^3}/kT \tag{12}$$

From Eq. 12 we may obtain directly

$$\log_e J_S = \log_e J_{S0} + \frac{\sqrt{e^3}}{kT}\sqrt{E} \tag{13}$$

Since $E$ is proportional to $V$, the anode potential, we may write also

$$\log_{10} J_S = \log_{10} J_{S0} + C\sqrt{V_p} \tag{14}$$

The constant $C$ includes the factor for conversion from natural to base-10 logarithms, as well as the other constants in Eq. 13 and the proportionality factor between $E$ and $V$.

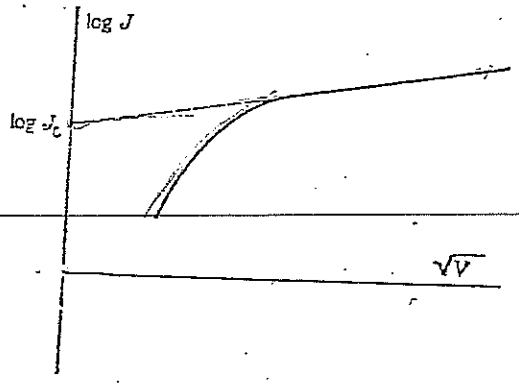


FIG. 6. SCHOTTKY PLOT. $V$—anode potential. $J_S$—saturation current density at cathode. $J_0$—saturation current density for zero field.

Equation 14 provides a means for obtaining values of $J_0$ with which to verify Richardson's equation and obtain values for $A$ and $w_0$. At each temperature, observations of $J$ are made for a series of values for $V$; these values then are plotted as shown in Fig. 6, with log $J$ as ordinates and $\sqrt{V_p}$ as abscissae. For smooth metal cathodes the upper (saturation) part of this plot is a straight line, thus verifying Schottky's theory. The $y$-intercept of its projection (broken line in Fig. 6) gives the value of log $J_0$, as shown by Eq. 14.

**11. Verification of Richardson's Equation.** The values of $J_0$ obtained in the manner described above may be checked against Richardson's corrected equation by substitution in Eq. 9. It is easier, however, first to convert Eq. 9 into the equation of a straight

line by dividing through by $T^2$, then taking logarithms of both sides, so as to give

$$\log_e \frac{J_0}{T^2} = \log_e A - \frac{w_0}{k}\frac{1}{T} \qquad (15)$$

Equation 15 indicates that a plot of $\log_e (J_0/T^2)$ against $1/T$, as represented in Fig. 7, should be a straight line, and experimental data do give a straight line over values of $J_0$ ranging from the smallest measurable current up to that corresponding to the highest workable temperature for each metal being studied. It is customary to express the work-function in electron-volts, and then to represent it by $\phi_0$. (See Sec. 6.) The exponent, $w_0/kT$, in Eqs. 9 and 15 then becomes

$$w_0/kT = 11{,}600\phi_0/T \qquad (16)$$

The value of $w_0$ (or $\phi_0$) for the metal studied is obtained from the plot of Fig. 7; its slope equals $w_0/k$, or $11{,}600\phi_0$.

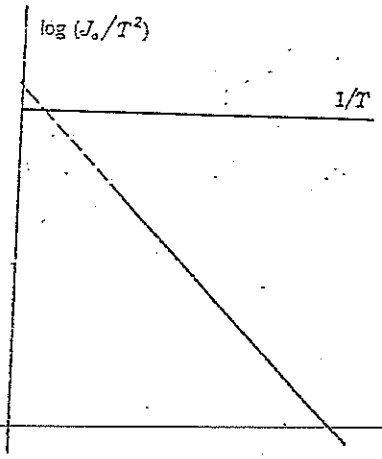


FIG. 7. RICHARDSON'S EQUATION PLOT. $J_0$—saturation current density (zero field). $T$—absolute temperature of cathode.

It would seem that this straight-line plot would be excellent proof for the correctness of Eq. 9 instead of Eq. 8, but this does not follow. Unfortunately, an almost equally straight line may be obtained for Eq. 8, by plotting $\log_e (J_0/\sqrt{T})$ against $1/T$. The slope of this line is somewhat greater than for the one plotted in Fig. 7, and represents $w'/k$. The effect of $T^2$, or of $\sqrt{T}$, is so small in comparison with that of $1/T$ in the exponent of $\epsilon$ that the experimental data do not decide between these two equations. Experimental evidence in favor of Eq. 9 is found, however, by comparing these values of the work-function with the photoelectric work-function given by Eq. 3. When photoelectric and thermionic currents are measured for the same surface the photoelectric work-function is found to agree better with the $w_0$ of Richardson's corrected equation than with the $w'$ of his original equation. The best validation for Eq. 9, however, is given by evidence to be considered later on.

12. Experiment 18. Thermionic Emission. To verify Richardson's equation for thermionic emission we must have a straight filamentary cathode of pure metal, and we must be able to measure its temperature. Furthermore, since the temperature of any filament falls off toward its ends, because of thermal conduction to its supports, accurate values for $A$ and $\phi_0$ cannot be obtained unless we are able to measure the electron current from only the central portion of the filament, where the temperature is uniform. Fortunately, a good

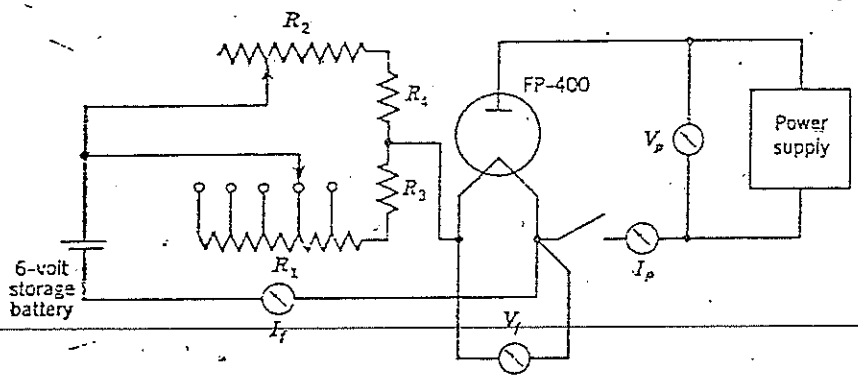


FIG. 8. CIRCUIT FOR EXP. 18.

$R_1$—2.5 ohms, tapped at 0.1, 0.2, 0.4, 0.6, 1.0, 1.5, and 2.0 ohms.
$R_2$—25 ohms, 50 watts.
$R_3$—1 ohm, 10 watts.
$R_4$—5 ohms, 10 watts.

check of Richardson's equation, and a fair value for $\phi_0$, may be obtained with a simple diode having a filament which is very long as compared with its diameter, and these requirements are met by the FP-400 Kenetron,* which has a pure tungsten filament, 1.25 inches long by 0.005 inch diameter, along the axis of a cylindrical anode about ½ inch in diameter.

The circuit is diagrammed in Fig. 8. Most trouble with this experiment arises from poor contacts in the heating circuit, at junction points, switch points, and rheostat sliders. All connections should be made with screw-clamps and binding-posts, not spring connectors. In place of the ordinary socket use screw-clamps for the tube terminals, or better still solder these connections. Storage-battery connections likewise should be screw-clamped, not made with battery clips. The main rheostat, $R_1$, should be of the heavy-duty, step-resistance type, with brass blocks for switch-points. Fine adjustment may be made

* Made by the General Electric Co.

with a tubular rheostat, $R_2$, in parallel with $R_1$. Keep a thin film of petroleum jelly over all surfaces of contact, especially on sliding surfaces of switches and rheostats.

Filament current and voltage, $I_f$ and $V_f$, must be read very closely, since they change by small amounts. Both meters should have large scales, finely graduated. The cathode temperature is determined from $I_f$, in the manner described in Part I. Do not exceed 4.25 volts for $V_f$, and do not hold it above 4.0 volts longer than needed to take observations. Correct $I_f$ for the voltmeter current, computed from $V_f$ and the resistance of this meter. A sequence of milliammeters is required to cover the range of values for $I_p$, the electron current, with enough overlap to keep all readings at least one-tenth of full-scale. Take 9 or 10 sets of observations, each set for a different value of $I_f$. Choose $I_f$ so that the resultant values of $I_p$ are spaced uniformly along a logarithmic scale. A suggested sequence of values for $I_p$ is 40, 20, 8, 4, 2, 0.8, 0.4, 0.2, 0.08, and 0.04 ma, measured at 100 volts anode potential. For each set take readings of $I_p$ for anode potentials of 300, 250, 200, 150, and 100 volts. Check $I_f$ and $V_f$ frequently to make sure they stay fixed throughout each set.

*Part I. Temperature.* The absolute temperature, $T$, may be determined from $I_f$ and the diameter, $d$, of the filament. According to work of Jones and Langmuir,* $T$ is a function of $I_f/d^{\frac{3}{2}}$, values of which are tabulated in their paper. Since $d$ must be taken from the tube maker's data and may be several percent in error, $T$ may be determined sufficiently well for this experiment from the following *empirical* equation.

$$T = 60.2 \sqrt{\frac{I_f}{d^{\frac{3}{2}}}\left(1 + 0.083 \times 10^{-3} \frac{I_f}{d^{\frac{3}{2}}}\right)} \quad ^\circ\text{K} \qquad (17)$$

Compute $I_f/d^{\frac{3}{2}}$ (amp/cm$^{\frac{3}{2}}$) from your data; then compute $T$ for each set.

*Part II. Schottky Effect.* Make a Schottky plot (Fig. 6) for each set of data and obtain $I_0$ from it. Semi-log plotting paper is very useful for these plots.

*Part III. Richardson's Equation.* (*a*) From the values of $T$ and $I_0$ obtained in Parts I and II make the plot of Fig. 7, with $1/T$ as $x$, $\log I_0/T^2$ as $y$. Draw the straight line which fits the plot best, and find $\phi_0$ from its slope. A value for $A$ may be computed from the

* H. A. Jones and I. Langmuir, *General Electric Review*, Vol. 30 (1927), pp. 310, 354. The tables are reproduced in E. L. Chaffee, *Theory of Thermionic Vacuum Tubes*, p. 100.

$y$-intercept, but this value will be poor, since the effective area of the cathode is uncertain.

(*b*) A shorter procedure, and one which has to be followed if the filament diameter is unknown, is to assume that $T$ is proportional to $I_f^{1/2}$ (see Eq. 17) and plot $1/I_f^{1/2}$ as $x$, $\log I_0/I_f$ as $y$. Over the range of temperature involved this assumption is reasonably good, and the plot should be a fairly straight line, in accord with Richardson's equation.

13. Work-Function Values. Values of $\phi_0$ for most metals lie between 5.0 ev, for platinum, and 1.8 ev, for cesium. The usefulness of a metal as a thermionic cathode, however, depends not only upon the value of $\phi_0$, but also upon the temperature at which the metal may be operated safely. In this respect tungsten is one of the best. Although its work-function is 4.52 ev, it may be operated without excessive evaporation above 2500 degrees K. Tungsten filaments are found in X-ray tubes, and in radio power tubes which are operated at high voltages.

For most purposes, various composite surfaces serve better than pure metals as thermionic cathodes. For example, thoriated tungsten, or tungsten covered with a monatomic layer of thorium, has a work-function much less than that for pure tungsten, less indeed than that for pure thorium, and operates best at a temperature for which its emission exceeds by one or two hundredfold that of pure tungsten at 2500 degrees K. Thoriated tungsten filaments are made from tungsten in which has been dissolved, in the molten state, 1 to 1.5 percent of thoria (thorium oxide). This filament is "activated" by heating it for a short time at about 2800 degrees K, then for 15 to 30 minutes at about 2100 degrees K. At the higher temperature thorium atoms are formed by reduction of some of the thoria; at the lower, these atoms diffuse to the surface, there to form an emissive layer. Thorium tends to evaporate from the surface, and must be replenished by diffusion of new atoms from the interior. At about 1900 degrees K these processes balance and a monatomic layer may be maintained. Above this temperature the evaporation is too rapid, and the diffusion is too slow if the temperature is much below this value.

The profound influence of a monatomic layer of thorium upon the emission from a tungsten cathode emphasizes the fact that thermionic emission is a *surface* phenomenon. Surface layers of other substances may produce comparable changes in the emission characteristics of cathodes, and many substances, for example oxygen, tend to lower or "poison" the emission. To avoid such poisoning, great care is taken to clean thoroughly all materials entering into the structure of an electron tube, and to evacuate it as thoroughly as possible. Gases

TABLE 8–3

THE RATIO $i/d^{3/2}$ AS A FUNCTION OF TEMPERATURE FOR A TUNGSTEN FILAMENT, WHEN $i$ IS THE CURRENT IN AMPERES AND $d$ IS ITS DIAMETER IN CENTIMETERS

| Temp., ° abs. | $i/d^{3/2}$, amp·cm$^{-3/2}$ | Temp., ° abs. | $i/d^{3/2}$, amp·cm$^{-3/2}$ |
|---|---|---|---|
| 293 | 0.0 | 2000 | 1022 |
| 300 | 3.727 | 2100 | 1119 |
| 400 | 24.67 | 2200 | 1217 |
| 500 | 47.62 | 2300 | 1319 |
| 600 | 75.25 | 2400 | 1422 |
| 700 | 108.2 | 2500 | 1526 |
| 800 | 148.0 | 2600 | 1632 |
| 900 | 193.1 | 2700 | 1741 |
| 1000 | 244.1 | 2800 | 1849 |
| 1100 | 301.0 | 2900 | 1961 |
| 1200 | 363.4 | 3000 | 2072 |
| 1300 | 430.9 | 3100 | 2187 |
| 1400 | 503.5 | 3200 | 2301 |
| 1500 | 580.6 | 3300 | 2418 |
| 1600 | 662.2 | 3400 | 2537 |
| 1700 | 747.3 | 3500 | 2657 |
| 1800 | 836.0 | 3600 | 2777 |
| 1900 | 927.4 | 3655 | 2833 |

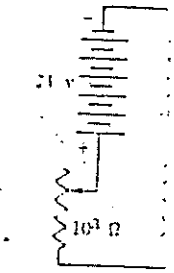


FIG. 8–19. ...son's equation

that it have a symmetric cathode. The most satisfactory tube to use is the GE-FP400, but one may also use the GE-FP84, RCA802, or 878 or the more common 6H6 or 76. With proper facilities, it is quite possible to construct a diode which is useful not only in studying Richardson's equation but also is suitable for investigating the distribution of velocities of the emitted electrons, and for making fairly accurate determinations of the constant $A$ and the $e/m$ ratio. A design of such a tube is shown in Fig. 8–19. Typical dimensions might be a filament diameter of 0.025 cm, an anode diameter of 2 cm, and a height of 1 cm. The two outer cylinders are three or four times longer than the anode, and are separated from the central section by not more than 1 to 2 mm. For such a tube the filament current is about 5 amp, the plate potential is 100 volts, and the plate current is about 1 to 2 ma.

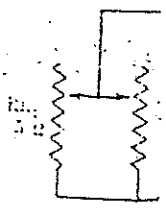

FIG. 8–

A typical circuit for obtaining emission current as a function of anode voltage for a series of filament temperatures is shown in Fig. 8–20. An experimental plot of $\log_{10}(i/T^2)$ against $1/T$ should be a straight line with a negative slope equal to $(e\Phi \log_{10} \epsilon)/k$. The intercept on the ordi-