Nuclear Counting

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Section 1
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I. ABSTRACT

II. INTRODUCTION

(((ROUGH)))

oldest radiation detector types in existence, intoduced by geiger and mueller in 1928.

simplicity, low cost, ease of operation

gas multiplication to greatly increase charge represented by the original ion pairs formed along the radiation track, this process is called Townsend discharge, like an avalanche of electrons

G-M tube uses high electric fields to enhance the intensity of each avalanche, where one avalanche can trigger another avalanche at a different position in the tube

at a critical value of the E field, each avalanche can create on average at least one other avalanche, at higher values the process becomes rapidly divergent within a very short time (too many shit avalanches)

the chain reaction will subside eventually due to collective effects from all these avalanches, this limiting point is always reached after the same number of avalanches is created, therefore all the collected pulsed from a G-M tube are of the same amplitude regardless of the number of original ion pairs (Radiation) that initiated the process.

The G-M tube cannot be used for radiation spectroscopy because the information about the energy of the incident radiation is lost.

III. THEORETICAL BACKGROUND

The Geiger-Mueller Counter

The Geiger-Mueller tube consists of a sealed tube with a window on the bottom face that allows particles into the tube, with a wire running down the middle inside. The tube is filled with usually a noble gas (due to its filled outer electron shell) and to a lesser degree, a quenching gas whose purpose will be explained shortly. An external circuit is usually part of the tube operation as an external quenching mechanism as well.

During operation, the tube and the wire are charged, to create an electric field inside the tube, with the tube being the cathode and the wire the anode. With the electric field inside the tube at a radius r given by:

$$E(r) = \frac{V}{rln(\frac{b}{a})} \tag{1}$$

where $a \equiv$ anode wire radius, $b \equiv$ cathode inner radius, and $V \equiv$voltage between the anode and cathode.

The process which the G-M tube detects ionizing radiation, is the process where the noble gases inside the tube are ionized producing an electron and a positive gas ion, which travel to the anode and cathode respectively. This electrical pulse in the unit can be detected as an ionization event, or a count from a radioactive source. For the pulses to be have a significant enough voltage to measure the G-M tube relies on the phenomenon of gas multiplication to amplify the effect of a single ionization event.

When an electron and ion pair are created, and in the large potential E-field of the G-M tube start to travel to their respective electrodes, the electron can hit another neutral noble atom producing another electron and ion pair, resulting in two electrons which may repeat this process toward the anode. This process is called a Townsend avalanche. However, the gas molecules will quickly return to their ground state, emitting a photon which can ionize another atom within the tube or liberate an electron from the tube wall, each of which result in another avalanche of electrons, which in turn, can create other avalanches within the G-M tube. This is the process of gas multiplication which can turn a single ionization event from a β-particle into many events, creating an electrical pulse strong enough to easily detect.

Since the time required to spread these avalanches is short and they only begin when the free electron and drifted to within a few mean free paths of the anode wire, the time it takes for the avalanches to grow in both directions along the wire only takes a small fraction of a microsecond. Since positive ions travel much slower than the electrons do, due to their mass, a high concentration of positive ions collect and sufficiently nullify the E field in the vicinity of the wire, and free electrons will no longer have a high enough ionization potential to create more avalanches until the cloud of positive ions travel outward to the cathode and equilibrium is restored to the tube, to a sufficient degree to create avalanches again during an ionization event. This period where the tube cannot create more avalanches is called the Dead Time.

The gasses inside the G-M tube is largely filled with a noble gas because of their neutral charge in inert nature, in our case the noble gas filling the G-M tubes is Neon. A secondary gas is also put into the tube for the purpose of internal quenching. After a discharge, as the positive ions are drifting toward the inner tube wall and becoming neutralized by combining with an electron from the cathode surface, if the difference of that atoms ionization energy and work function of the cathode is greater than the work function of the cathode surface, there is a small chance that an extra electron can be liberated which can cause a Townsend avalanche leading to the G-M discharging again. To fix this there is the process of quenching, externally through a circuit which happens after discharging, the circuit reduces the high voltage applied to the tube for a small fixed time after the pulse using a capacitor circuit, this stops the process of gas multiplication so secondary avalanches cannot be formed. There is also the method if internal quenching, which uses a quench gas (as mentioned previously) in lower concentrations with the primary noble fill gas (about 5-10%) which has a lower ionization potential, its purpose is to stop multiple pulsing through charge transfer collisions. As the positive ions of the primary gas, while drifting toward the cathode, collide with the quench gas molecules, because of the lower ionization energy the positive charge is transferred to the quench gas molecule and the primary gas atom becomes neutral. If the concentration of the quench gas is sufficient, only quench gas molecules will be neutralized at the tube surface and because of the lower ionization energy the probability of dissociation is larger than the liberation of a free electron. In our case, a Halogen gas is used as the quench gas.

When looking at the charge on the tube versus the count rate, it is obvious that when the charge is below a certain threshold that there will be no recorded count rate, when the count are first recorded this is called the starting voltage, and the count rate increases rapidly as the voltage is increased up to a point where the rate plateaus. Increases the voltage doesn't change the count rate by any significant amount. There will be a point though, a critical voltage value above this where the G-M tube will continuously discharge, which may damage the tube. This plateau generally has a slope of 2-10% per 100V.

Determination of the Dead Time

If we assume a β-particle emitter gives N counts per second, with zero dead time, and n counts per second with a counter with a dead time of T seconds. So the counter cannot detect nT counts within this period, so the dead time is equivalent to:

$$N_i - n_i = N_i n_i T \tag{2}$$

Using two sources, we can build up a system of equations for the count rates with each source individually and one with the two sources measured together. Solving for T we can find T to be:

$$T \approx \frac{n_1 + n_2 - n_3}{2n_1 n_2} \tag{3}$$

with a factor of $O(nT)^2$ to be omitted, and it is assumed at the counts are generally equally space in time.

Random Nature of Radioactive Decay

$$t = \frac{n - m}{\sigma} \tag{4}$$

$$P_o(t)dt = \frac{e^{-\frac{t^2}{2}} dt}{\sqrt{2\pi}} \tag{5}$$

IV. EXPERIMENTAL DESIGN AND PROCEDURE

Characteristics of Geiger Tubes

A single source of Chlorine-36 was placed under the Geiger tube and enclose within a lead stack to minimize background the effect of background radiation. The Geiger tube was attached to a power supply as well as a voltage measuring device and a counting device. Allowing the operating voltage to be determined and the count and voltage to be read by the system. A computer was attached to the voltage measuring and counting devices, allowing much of the measurements to be automated. To determine the characteristics of the two supplied Geiger tubes, LND 712 and LND 72314 models, we loaded a program that would measure the total count over a specified time interval. We set the time interval for 60 seconds and measured the total at a number of different operating voltages. These voltages ranged from 300-650V for the LND 712 model and from 300-700V for the LND 72314 model, in increments of 25V. From this data we were able to plot a graph relating the radiation count to the operating voltage and thus decide on which tube and voltage would be suitable for the subsequent tests. The subsequent tests were completed using the LND 72314 model at an operating voltage of 575V.

Measuring the Dead Time

To measure the dead time we used a dual source of β Chlorine-36 in the same arrangement as the single source for the previous test. Using the LND 72314 Geiger tube at an operating voltage of 575V we measured the amount of radiation detected in 60 second intervals. We took four separate readings for the background, first source, second source and both sources. The test was repeated 5 times to increase reliability.

Statistics of Radioactive Decay

For statistical analysis of the regularity of the radioactive decay we performed a number of measurements of the number of counts in a small time interval. The time interval used was calculated according to the formula

$$T_i = T_0 \cdot \frac{20}{N_0} \tag{6}$$

where T_0 and N_0 are the time interval and count from the first experiment. By using this time interval the expected value for the radiation emitted is 20 counts. Thus allowing us to test the variance from this figure. Substituting values into eq. 6

$$T_i = \frac{60 \times 20}{4629} = 0.259s$$

In each test we measured the count over the time interval N times, where N = 10,100,300,500.

Range of Beta Particles in Aluminium

To measure the range of the β particles through aluminium we performed measurements of the amount of radiation detected in 60 seconds for various thicknesses of aluminium shielding. The shields were placed above the source in the stack, ensuring that when changing the shield the Geiger tube wasn't exposed to the source alone. This is to eliminate the possibility of hysteresis errors.

V. ANALYSIS

Characteristics of Geiger Tubes

The relations between applied voltage and radiation count for the LND 712 and LND 72314 Geiger tubes are plotted in figures 1 and 2 respectively.

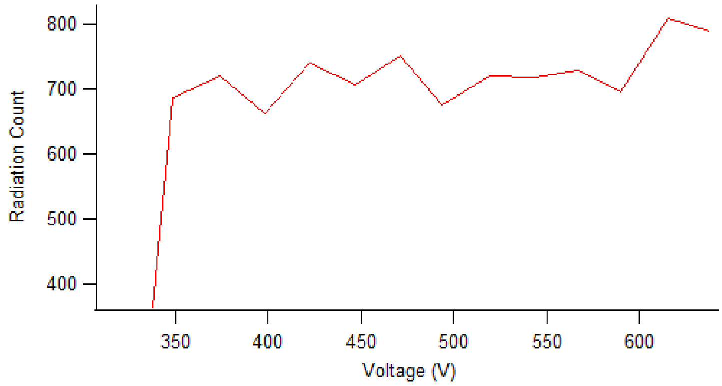

FIG. 1: Radiation count versus the applied voltage for an LND 712 model Geiger tube

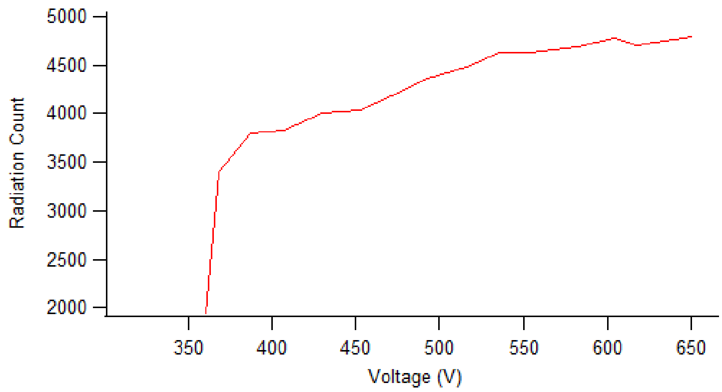

FIG. 2: Radiation count versus the applied voltage for an LND 72314 model Geiger tube

The characteristic data for each Geiger tube is given in table V 16. It was from this data that the decision on the model and operating voltage of the tube was decided for subsequent tests. It can be seen that the slope in the LND 712 model is smaller than that of the LND 72314. Although this is a preferred quality, it was decided that the higher count rate exhibited by the LND 72314 would be more useful than closer to ideal slope, because of this the decision was made to use this model.

Geiger Tube Model	Starting Voltage (V)	Slope (%per V)
LND 712	368	0.0499
LND 72314	349	0.115

TABLE I: Characteristic data of Geiger Tubes

Sample Calculation for slope

$$Slope = \frac{(\frac{\Delta C}{\Delta V})}{mean\ count\ rate} \times 100\% perV$$

$$\frac{\Delta C}{\Delta V} = \frac{C_2 - C_1}{V_2 - V_1} = \frac{4792 - 3395}{651 - 368} = 4.94$$

$$Slope = \frac{4.94}{4305} \times 100 = 0.115\% perV$$

Measuring the Dead Time

The average values for the count rate of the sources are given below, the uncertainty is taken to be one standard deviation. n_1 represents the difference between the first source count rate and the background count rate, n_2 represents the difference between the second source count rate and the background count rate and n_3 represents the difference between the combined count rate and the background count rate.

$$n_1 = 1472 \pm 108$$
$$n_2 = 6490 \pm 77$$
$$n_3 = 7427 \pm 90$$

$$\Delta(n_1 + n_2 - n_3) = \sqrt{108^2 + 77^2 + 90^2} = 160$$
$$\frac{\Delta T}{T} = \sqrt{\left(\frac{160}{535}\right)^2 + \left(\frac{77}{6490}\right)^2 + \left(\frac{108}{1472}\right)^2} = 0.31$$
$$T = \frac{n_1 + n_2 - n_3}{2n_1 n_2} = \frac{1472 + 6490 - 7427}{2 \times 1472 \times 6490} = (2.8 \pm 0.9) \times 10^{-5} s$$

Statistics of Radioactive Decay

10 Trials

A histogram showing the relative frequency of readings for a sample size of 10 is given in figure 3.

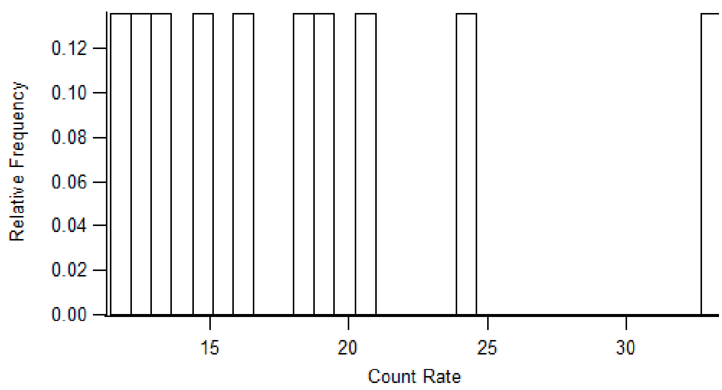

FIG. 3: Relative frequency of count rates for n = 10.

Statistical data for this test is as follows

Mean Count Rate	18
Standard Deviation	6.63
Mean Deviations	4.8

$$1. \quad Standard\ Deviation = \sqrt{mean\ count}$$

$$\sqrt{mean\ count} = \sqrt{18} = 4.24$$
$$\%_{diff} = \frac{4.24 - 6.63}{6.63} \times 100 = -36\%$$

with a difference of 36%, this statement lacks validity when applied to this particular test.

$$2. \quad Mean\ Deviation = \tfrac{4}{5}\ Standard\ Deviation$$

$$\frac{4}{5}\sigma = \frac{4}{5} \times 6.63 = 5.3$$
$$\%_{diff} = \frac{5.3 - 4.8}{4.8} \times 100 = 10\%$$

in this case the standard deviation is found to be 10% greater than the mean deviation. This value indicates conformity with the statement, although the sample size is still too small to draw useful conclusions.

$$3. \quad \tfrac{1}{3}\ of\ deviations\ exceed\ one\ standard\ deviation$$

From this data group it is found that 2 out of 10 of the deviations exceed that of the standard deviation. Calculating the % difference between the statement and the findings

$$\%_{diff} = \frac{\frac{1}{5} - \frac{1}{3}}{\frac{1}{3}} \times 100 = -40\%$$

again, with a difference of 40% below the expected values the statement lacks validity when applied to a sample of this size.

$$4. \quad \tfrac{1}{20}\ of\ deviations\ exceed\ two\ standard\ deviations$$

It was found that 1 of the 10 deviations exceeded two standard deviations. This value is twice of that expected from the statement.

100 Trials

A histogram showing the relative frequency of count rates for 100 samples is given in figure 4.

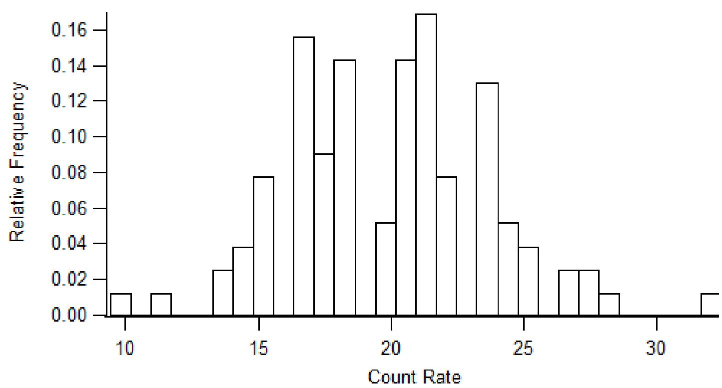

FIG. 4: Relative frequency of count rates for N = 100

Statistical data for this experiment is given below

Mean Count Rate	19.6
Standard Deviation	3.87
Mean Deviation	3.12

5. Standard Deviation $= \sqrt{mean\ count}$

$$\sqrt{mean\ count} = \sqrt{19.6} = 4.42$$
$$\%_{diff} = \frac{4.42-3.87}{3.87} \times 100 = 14.2\%$$

a difference of 14%, whilst still not ideal, is far better than previous tests and suggests that a higher sample size may lead to greater agreement between the statements and the results

6. Mean Deviation $= \frac{4}{5}$ *Standard Deviation*

$$\frac{4}{5}\sigma = \frac{4}{5} \times 3.87 = 3.096$$
$$\%_{diff} = \frac{3.096-3.12}{3.12} = -0.77\%$$

a difference of 0.77% below the expected value shows results that strongly agree with the statement. Adding validity to the hypothesis that agreement between statements and results will increase with sample size.

7. $\frac{1}{3}$ *of deviations exceed the standard deviation*

It is found that 26 of the 100 deviations exceed the standard deviation. When compared to the expected value of $\frac{1}{3}$

$$\%_{diff} = \frac{\frac{26}{100} - \frac{1}{3}}{\frac{1}{3}} \times 100 = -22\%$$

although this value has a smaller difference to the expected value than the previous test. The size of the difference still renders it inconclusive.

8. $\frac{1}{20}$ *of deviations exceed two standard deviations*

It is found that 4 of the 100 deviations exceed two standard deviations. When compared to the expected value of $\frac{1}{20}$

$$\%_{diff} = \frac{\frac{4}{100} - \frac{1}{20}}{\frac{1}{20}} \times 100 = -20\%$$

the same is true in this case; although the congruence in this test is better than that of the smaller sample size it is still inconclusive.

200 Trials

A histogram showing the relative frequency of count rates for 200 trials is given in figure 5.

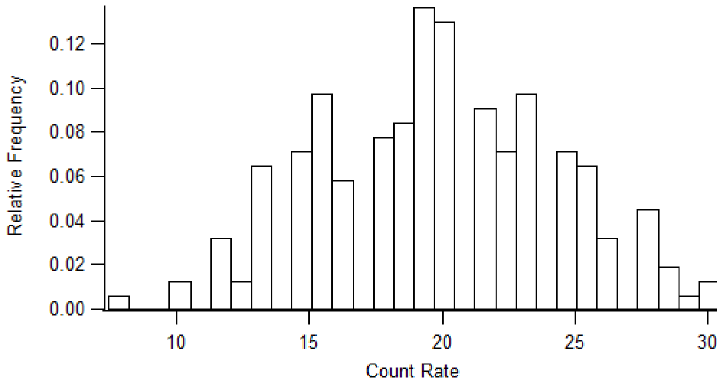

FIG. 5: Relative frequency of count rates for N = 200

Statistical data for this experiment is given below

Mean Count Rate	19.4
Standard Deviation	4.50
Mean Deviation	3.64

9. Standard Deviation = $\sqrt{mean\ count}$

$$\sqrt{mean\ count} = \sqrt{19.4} = 4.40$$
$$\%_{diff} = \frac{4.40-4.50}{4.50} \times 100 = -2.2\%$$

This difference is less than both of the previous tests, in which smaller sample sizes were used. With a difference of 2.2% there is a strong indication of the validity of the statement, as well as the hypothesis that agreement will increase with sample size

10. Mean Deviation = $\frac{4}{5}$ Standard Deviation

$$\frac{4}{5}\sigma = \frac{4}{5} \times 4.50 = 3.60$$
$$\%_{diff} = \frac{3.60-3.64}{3.64} = -1.1\%$$

a difference of 1.1% below the expected value shows results that strongly agree with the statement. Although the difference in this value is greater than that for the previous test, the magnitude of the difference is small enough to be accounted for by the random nature of the readings.

11. $\frac{1}{3}$ of deviations exceed the standard deviation

It is found that 70 of the 200 deviations exceed the standard deviation. When compared to the expected value of $\frac{1}{3}$

$$\%_{diff} = \frac{\frac{70}{200} - \frac{1}{3}}{\frac{1}{3}} \times 100 = -5\%$$

this value is considerably smaller than those found in tests with smaller sample size. The size of the value suggests agreement with the statement for trials with a greater sample size.

12. $\frac{1}{20}$ of deviations exceed two standard deviations

It is found that 6 of the 200 deviations exceed two standard deviations. When compared to the expected value of $\frac{1}{20}$

$$\%_{diff} = \frac{\frac{6}{200} - \frac{1}{20}}{\frac{1}{20}} \times 100 = -40\%$$

the agreement in this test is considerably worse than in previous tests. Due to the general trend towards greater agreement for larger sample sizes, however, I would put this down to the random nature of the readings.

300 Trials

A histogram showing the relative frequency of count rates for 300 samples is given in figure 6.

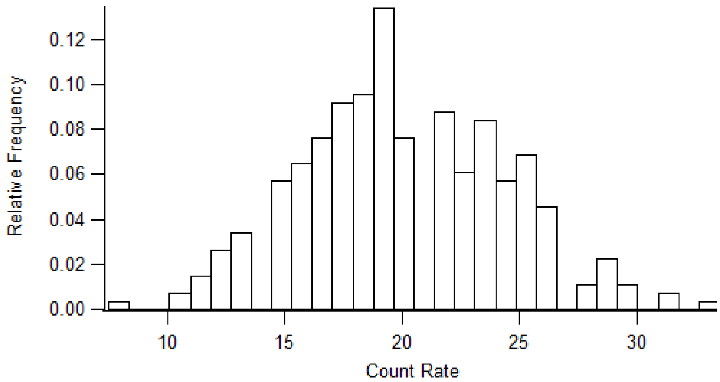

FIG. 6: Relative frequency of count rates for N = 300

Statistical data for this experiment is given below

Mean Count Rate	19.6
Standard Deviation	4.39
Mean Deviation	3.55

13. Standard Deviation $= \sqrt{mean\ count}$

$$\sqrt{mean\ count} = \sqrt{19.6} = 4.42$$
$$\%_{diff} = \frac{4.42-4.39}{4.39} \times 100 = 0.68\%$$

these are very close to each other and are therefore consistent with the statement. This value is also less than all previous tests using smaller sample sizes, consistent with the hypothesis that a greater sample size will increase agreement between results and statements.

14. Mean Deviation $= \frac{4}{5}$ *Standard Deviation*

$$\frac{4}{5}\sigma = \frac{4}{5} \times 4.39 = 3.512$$
$$\%_{diff} = \frac{3.512-3.55}{3.55} = -0.01\%$$

a difference of 0.01% below the expected value shows results that strongly agree with the statement. Adding validity to the hypothesis that agreement between statements and results will increase with sample size.

15. $\frac{1}{3}$ *of deviations exceed the standard deviation*

It is found that 115 of the 300 deviations exceed the standard deviation. When compared to the expected value of $\frac{1}{3}$

$$\%_{diff} = \frac{\frac{115}{300} - \frac{1}{3}}{\frac{1}{3}} \times 100 = 15\%$$

although this value has a smaller difference to the expected value than the previous test. The size of the difference still renders it inconclusive.

16. $\frac{1}{20}$ *of deviations exceed two standard deviations*

It is found that 9 of the 300 deviations exceed two standard deviations. When compared to the expected value of $\frac{1}{20}$

$$\%_{diff} = \frac{\frac{9}{300} - \frac{1}{20}}{\frac{1}{20}} \times 100 = -40\%$$

This value is equal to that of the test with N = 200. Both values have too great of a difference to the expected value to allow us to conclude agreement with the statement. These values are also inconsistent with the hypothesis that a greater sample size will lead to greater agreement.

500 Trials

A histogram showing the relative frequency of count rates for 500 samples is given in figure 7.

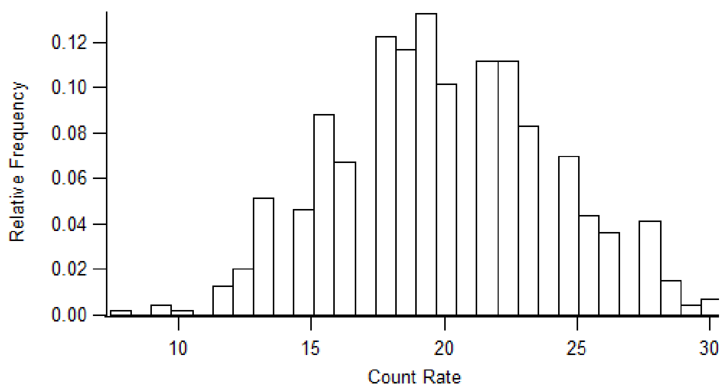

FIG. 7: Relative frequency of count rates for N = 500

Statistical data for this experiment is given below

Mean Count Rate	19.5
Standard Deviation	4.09
Mean Deviation	3.32

17. *Standard Deviation* $= \sqrt{mean\ count}$

$$\sqrt{mean\ count} = \sqrt{19.5} = 4.42$$
$$\%_{diff} = \frac{4.42 - 4.09}{4.09} \times 100 = 8.06\%$$

the difference in these values is significantly larger than that of the previous two tests. Although the difference is still small enough to indicate agreement between the statement and the results, it is inconsistent with the hypothesis of agreement increasing with sample size

18. *Mean Deviation* $= \frac{4}{5}$ *Standard Deviation*

$$\frac{4}{5}\sigma = \frac{4}{5} \times 4.09 = 3.27$$
$$\%_{diff} = \frac{3.27 - 3.32}{3.32} = -1.5\%$$

a difference of 1.5% below the expected value shows results that strongly agree with the statement. The difference is, however, greater than that for tests of smaller sample sizes.

19. $\frac{1}{3}$ of deviations exceed the standard deviation

It is found that 174 of the 500 deviations exceed the standard deviation. When compared to the expected value of $\frac{1}{3}$

$$\%_{diff} = \frac{\frac{174}{500} - \frac{1}{3}}{\frac{1}{3}} \times 100 = 4.4\%$$

the difference between expected and measured values in this test is conclusive with the statement. It is also smaller than previous tests using smaller sample sizes.

20. $\frac{1}{20}$ of deviations exceed two standard deviations

It is found that 20 of the 500 deviations exceed two standard deviations. When compared to the expected value of $\frac{1}{20}$

$$\%_{diff} = \frac{\frac{20}{500} - \frac{1}{20}}{\frac{1}{20}} \times 100 = -20\%$$

This value is equal to that of the test with N = 100. Both values have too great of a difference to the expected value to allow us to conclude agreement with the statement. There is also not enough evidence to show that the agreement between this statement and the measured results is increasing with sample size.

Range of Beta Particles in Aluminium

The graph showing the count rate versus thickness of shielding is given in figure 8. From this two linear plots are seen to be intersecting. The rightmost plot is that of the background radiation, whereas the leftmost plot is that of the β radiation. The point at which these plots meet is the range in aluminium for β radiation from a single Chlorine-36 source.

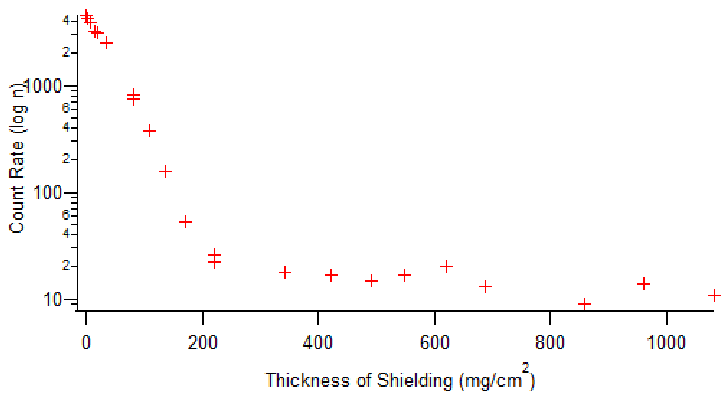

FIG. 8: Count rate versus thickness of aluminium shielding

By observation the range is determined to be $R = 200 \pm 20 mg/cm^2$. With the uncertainty an estimated value, allowing for the large inaccuracies associated with measuring a value by eye. When this value is used in comparison on the curve in Halliday, relating energy and range in Al, the energy of β Chlorine-36 radiation is determined to be $E = 0.6 \pm 0.5 MeV$. The accepted value for the energy of beta radiation as given in brown is 0.709MeV. Our value is relatively consistent with this with a percentage difference of 15%.

VI. CONCLUSION

Although we found the LND 72314 Geiger tube to be sufficient for our needs and found the increased count number to be beneficial, we determined when analysing the data that the LND 712 would have been the better choice. This is because of the lower slope value found in the plateau, allowing for more accurate readings with less impact by voltage fluctuations.

The measured results for the statistics of radioactive decay were consistent with the statements provided by brown. It was also apparent that for a larger sample size, the agreement between theory and experiment was increased. There were, of course, exceptions to this found in the experiment but nothing that would indicate anything other than the random nature of radioactive emittance.

The value for the energy of β radiation from a Chlorine-36 source, as determined by our experiment was found to be consistent with that of the expected value.

VII. REFERENCES