The Speed of Light

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Section 1
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I. ABSTRACT

It is classically known that light propagates at a finite velocity. In our experiment, we make use of a fast spinning mirror and laser pulses to quantify the time difference of propagation along two different path lengths. Through variations of the optical systems, it can be shown that there is a geometrical relationship in the optical system that allows the speed of light to be determined. Through the use of these relationships we have managed to determine the speed of light from these geometrical relationships alone. Using two different methods we determined values for $c = (2.96 \pm 0.05) \times 10^8 ms^{-1}$ and $c = (2.99 \pm 0.0045) \times 10^8 ms^{-1}$.

II. INTRODUCTION

The speed of light, c, which comes from the Latin word celer, meaning fast, is in the modern day a well known and used constant, so much so that the meter was redefined in 1983 at the 17^{th} Conférence Générale des Poides et Mesures such that the speed of light in a vacuum was set to exactly $c = 2.99792458 \times 10^8 \frac{m}{s}$. Throughout history the topic of light and its speed has been widely debated, and many of its properties are still only being understood.

Some of the first accurate measurements were carried out by Fizeau in 1849 using a rotating toothed wheel and a distant mirror, and from the rotational speed of the wheel and the observed returning pules, or obstruction the speed of light could be calculated and Fizeau arrived at the value of $3.15300 \times 10^8 \frac{m}{s}$ for the speed of light. This experiment was improved by Foucault and then Michelson in the 1920s by using rotating and fixed mirrors. We will be using a similar set-up in the first part of this lab.

In this experiment, by reflecting a laser beam off the rotating mirror, then back onto itself though a total optical distance of about 40m into a travelling microscope from which we measured the relative displacement of the beam image S′′. Due to the difference in angular position of the rotating mirror from first reflection and the returning reflection, the image location S′′ differs by some amount according to the rotational speed of the spinning mirror. By varying the speed of the rotating mirror we measured various displacements and using the equations derived in the succeeding section we were able to calculate a value for the speed of light in air, c_{air}.

For the second part of this lab we used another, more direct method of directly measuring the time difference between two, different, *known* optical path lengths of light from a pulsed laser measuring the two pulses on a digital storage oscilloscope. We took measurements for several optical path lengths between 40-60m using a system of mirrors, lenses, and a retro-reflector.

III. THEORETICAL BACKGROUND

In the experiments to determine the speed of light, the light beams are propagated through open air. Air, assuming ideal circumstances, has an optical index of approximately 1.0003. The optical index of the medium of propagation affects the velocity of propagation through the system in a manner inversely proportional to the optical index of the material (Eq.1).

$$v_m = \frac{c}{n_m} \tag{1}$$

For the medium in the experiment, air, the documented value of the index of refraction is approximately 1.0003 [Hecht, 2002]. Under ideal circumstances, the impact on the actual speed of light would be about 0.3%. It can safely be assumed that this factor plays a minimal role considering the expected errors in path measurements in the optical system.

In both parts of the experiment, the basic equation of distance versus time is used in determining the speed of light.

$$v_c = \frac{d}{t} \tag{2}$$

The first part of the experiment, the Michelson-Foucault method, relies on a rotating mirror, M_1 which periodically aligns with the remaining optical path, $L_2M_2M_3$, to produce a return beam in the travelling microscope, TM, see Fig.1. In the time for the light to propagate from M_1 to M_3 and back to M_1, the mirror M_1 will have rotated through an angle θ before retuning the beam to the travelling microscope. The value of θ is of course dependant on a combination of the path length between mirror reflections and the frequency which the mirror is rotating, f_m. Since the mirror is double sided, under one revolution of the motor, the mirror actually undergoes 2 full passes of the system, double the reported frequency of the motor.

$$\theta = (2\pi f_m)\left(\frac{2d_{m1m3}}{c}\right) \tag{3}$$

With the mirror having rotated through θ, the returning beam be deviated from its initial path. Through the travelling microscope, this deviation can be measured as the height, d, of a right angle triangle formed by the rotating mirror, the initial beam position, and the deviated beam position. Using some basic trigonometry (with a small angle approximation), the angle of deviation will allow a determination of the deviation from the incident beam.

$$d = \tan\theta d_{L1M1} = \theta d_{L1M1} \tag{4}$$

Which combined with the earlier determined deviation angle, Eq.3, returns a simple relationship which allows the speed of light to be determined using a relationship of motor speed and deviation length:

$$d = (2\pi f_m)\left(\frac{2d_{m1m3}}{c}\right)d_{L1M1} \tag{5}$$

The second part of the experiment, pulsed laser method, relies on short light pulses from a controlled source. The use of the 5.0m focal length lens at 5.0m collimates the outgoing beam, no convergence/divergence, such that no matter the path length, the beam returning through the lens to the detector is focused and detectable.

By changing the distance between the second mirror and the corner cube reflector, the time gap between the initial pulse and the return pulse may be changed and measured using the oscilloscope.

$$\Delta t = 2\left(\frac{d_{splitter-cornercube} + d_{splitter-mirror}}{c}\right) \tag{6}$$

IV. EXPERIMENTAL DESIGN AND PROCEDURE

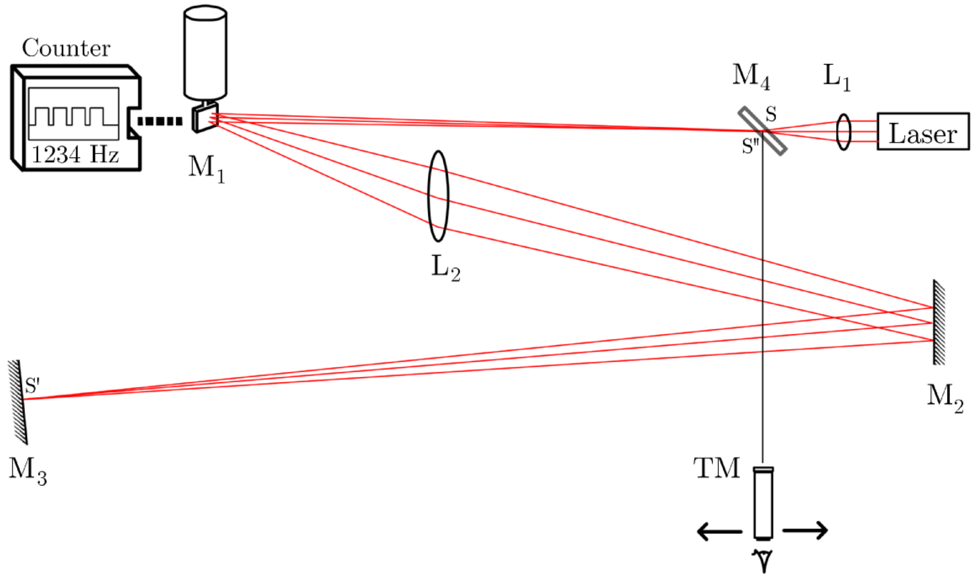

FIG. 1: Sketch of the Michelson-Foucault variation experiment to measure the speed of light, a laser which passes through lenses L_1 with focal length $f_1 = 17$cm and L_2 with $f_2 = 5$m, flat rotating mirror M_1, flat fixed mirrors M_2 and M_3, and half-silvered mirror M_4 and a travelling microscope TM to measure the position of image S'' for several different rotational speeds for M_1. Counter will capture the pulses as M_1 rotates and blocks the light from the laser giving a frequency of twice the actual rotational frequency of M_1.

We set up our equipment as seen in Fig.1. We set up the laser and L_1 such that image S is produced on M_4, and set up L_2 (with a focal length, $f_2 = 5$m), such that the distance from S to M_1 to L_2 is $2f_2 = 10$m. This means the image S' will also be created a length of $2f_2$ away from L_2 due to the Thin lens formula $\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}$. M_1 was kept stationary and rotated manually to a fixed point such that it lined up the reflected beam with with L_2; M_1 stayed in this position until the other mirrors are set up.

Similarly the distance travelled by the light from L_2 to mirrors M_2 and M_3 also needed to be $2f_2$ so the light beam is reflected back upon itself after producing a sharp image, S', on M_3 so that the reflected beam would travel back though L_2 and be focussed onto M_4.

The lengths between M_4 and M_1, M_1 and L_2, L_2 and M_2, and lastly M_2 and M_3 were measured using a long plastic tape measure, using 2 people, one of each end. We made sure to pull the tape taught as it would sag in the middle over the long distances(≈ 5m) we were taking measurements for. We estimate that we placed the mirrors and lenses within ± 1cm of uncertainty for the 4 measurements we took due to the bending of the tape measure.

Next the counter photocell was placed behind M_1 such that when M_1 was spinning, light would pass by when M_1 was parallel to the light beam and be collected in the counter. The counter analyses the period of light intensity and displays a frequency in Hz, which is 2 times the actual rotational frequency of M_1. With the mirror M_1 rotating, an image S'' is reflected off of M_4 into the travelling microscope which can be translated horizontally to measure the displacement of S'' as the rotational speed of M_1 changes. Two images are seen though the travelling microscope, one next to the other, as the beam reflects off of the front *and* internal surfaces of M_4. One image was chosen and that one was used to measure the relative displacement between trials for the rest of the experiment.

10 positions for S'' were measured and recorded along with the displayed rotational frequency, between 100Hz and 1000Hz in evenly spaced intervals of 100Hz. The counter fluctuated ± 1Hz during operation so we assumed a 2Hz uncertainty in this measurement. Care was taken to avoid gear slipping with the translation apparatus on the travelling microscope so that readings on the attached calliper were accurate. Care was also taken to monitor the counter while the increasing the voltage to M_1, because near 700Hz the displayed frequency inexplicably dropped, the photocell needed to be readjusted so that it accurately read the incoming intensity and the counter displayed the correct frequency.

After we completed the 10 trials, we repeated another 10 trials, this time ranging from 150Hz to 1050Hz, also in evenly spaced intervals of 100Hz, recording the measured rotational frequency from the counter and the S'' spot displacement for each.

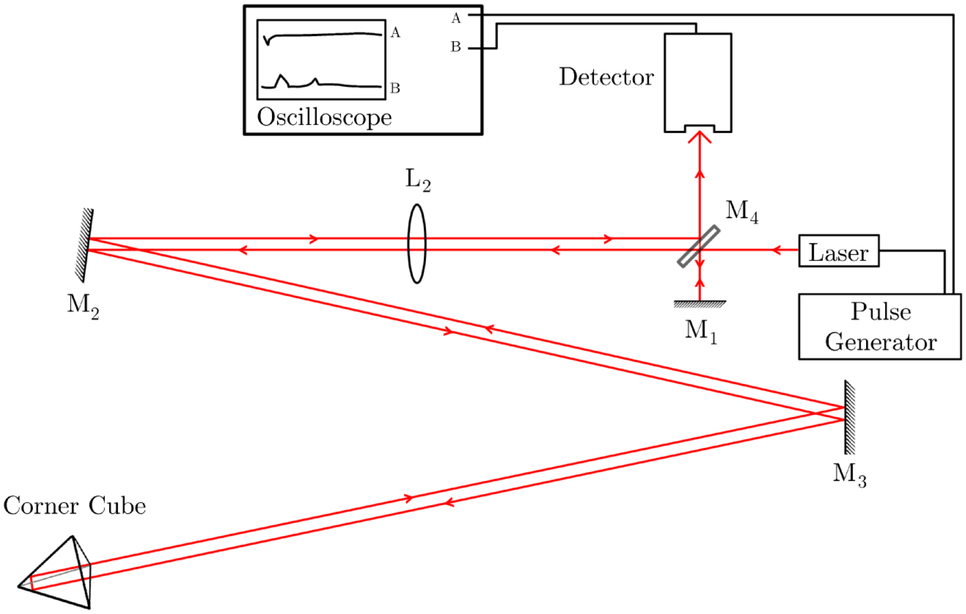

FIG. 2: Pulse generator pulses the laser which follows the optical path outlined, one split up into the detector while the other continues and is reflected back on itself with the corner cube where it is also collected by the detector after some time t. The path differences of the split pulses are measured for several different path length by varying the distance of the corner cube in the optical path.

We set up our equipment as shown in Fig.2. The laser which is focussed about 30cm in front of the laser, is a red light emitting semiconductor laser can have its power modulated by an applied voltage, directs a beam down towards a half silver mirror M_4. The beam is split, reflecting to wards M_1 and then into the fast photo-diode and is measured by the oscilloscope; the distance from M_4 to M_1 was measured using a ruler and recorded. The other beam continues to L_2 with $f_2 = 5$m. We placed L_2 5m away from M_4 such that the unfocussed beam becomes collimated as it travels to M_2, M_3, and lastly the corner cube which reflects the beam back exactly on its previous trajectory. Lengths M_4 to L_2, L_2 to M_2, and M_2 to M_3 were measured using the tape measure and recorded. Care was taken to keep the tape measure from sagging to get an accurate measurement. We estimate that we placed the mirrors and lenses within $\pm$1cm of uncertainty for the measurements we took due to the bending of the tape measure.

As the beam travels back off of M_3 and M_2 and then through L_2, it focuses back onto M_4 and bounces into the the fast photo-diode and is measured by the oscilloscope. Using the oscilloscope we measured the times between the incoming and the reflected beam and recorded the time as well as the distance to the corner cube, allowing us to measure the path difference and thus, able to measure the speed of light directly. We did these measurements for 3 different distances by varying the position of the corner cube in the optical path, measuring and recording the distance from M_3 to the corner cube as well as the time difference of the two beams by reading the data on the oscilloscope. We made sure to make consistent measurements using the cursors on the oscilloscope, measuring the distance from peak to peak on the displayed waveform, where each peak corresponds to the two detected beams, the split beam and the reflected.

V. ANALYSIS

A. A. Michelson-Foucault Method

The graphs showing measured values of displacement against frequency are given in Fig.3. The uncertainties in displacement have been omitted from the graph as they are too small to see. The uncertainties in frequency ($\pm 2Hz$) are shown in the error bars.

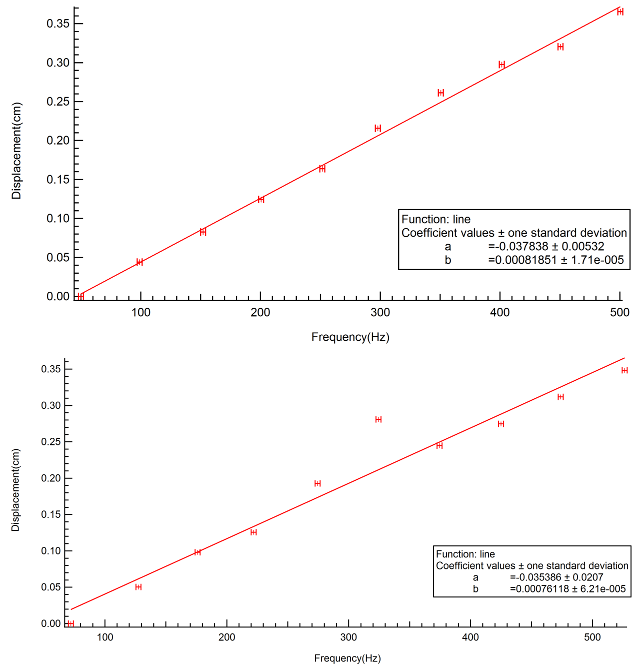

FIG. 3: Displacement measurements $(x - x_0)$ against measured frequency of oscillating mirror

Rewriting Eq.5 given in the theory section to give displacement as a function of frequency gives:

$$d = (\frac{4\pi \times 2 \times d_{m_1 m_3} d_{L1M1}}{c}) f_m \tag{7}$$

The factor of 2 in this equation comes from the fact that the light beam travelled each measured distance twice during its path through the apparatus. From this graph it can be seen that the slope of the best fit line will be equal to the coefficient of f:

$$S = \frac{d(d)}{df} = \frac{4\pi \times 2 \times d_{m_1 m_3} d_{L1M1}}{c} \tag{8}$$

And solving for c using the two values of slope given in the graphs

$$c = \frac{4\pi \times 2 \times d_{m_1 m_3} d_{L1M1}}{\frac{d(d)}{df}}$$

$$c_1 = \frac{4\pi \times 2 \times 6.6 \times 13.4}{0.0008185 \times 10^{-2}} = 2.96 \times 10^8 ms^{-1}$$

$$c_2 = \frac{4\pi \times 2 \times 6.6 \times 13.4}{0.00076118 \times 10^{-2}} = 2.92 \times 10^8 ms^{-1}$$

Calculating uncertainties in experimentally determined value for $c_1 and c_2$

$$\%e_{S_1} = \pm 100 \times \frac{1.71 \times 10^{15}}{0.00081851} = \pm 2.1\%$$

$$\%e_{d_{m_1 m_3}} = \pm 100\ times \frac{0.02}{6.6} = \pm 0.3\%$$

$$\%e_{d_{L1M1}} = \pm 100\ times \frac{0.02}{13.4} = \pm 0.3\%$$

$$\%e_{c_1} = \sqrt{2.1^2 + 0.3^2 + 0.3^2} = \pm 2.0\%$$

$$\%e_{S_2} = \pm 100 \times \frac{6.21 \times 10^{-5}}{0.00076118} = \pm 8.2\%$$

$$\%e_{c_2} = \pm\sqrt{8.2^2 + 0.3^2 + 0.3^2} = \pm 8.2\%$$

So our values of $c_1 and c_2$ with uncertainties are

$$c_1 = (2.96 \pm 0.05) \times 10^8 ms^{-1}$$
$$c_2 = (2.92 \pm 0.24) \times 10^8 ms^{-1}$$

Finding the average value for c weighting the values inversely proportionally to the associated error

$$\bar{c} = \frac{\Sigma \frac{c_n}{e_n^2}}{\Sigma \frac{1}{e_n^2}}$$

$$\bar{c} = \frac{\frac{2.96}{0.05^2} + \frac{2.9}{0.24^2}}{\frac{1}{0.05^2} + \frac{1}{0.24^2}} = (2.96 \pm 0.05) \times 10^8 ms^{-1}$$

Instrumental errors affected the uncertainties greatly in this experiment, especially those associated with reading the frequency. The frequency shown on the device was found to deviate by ±2Hz with each reading as it wouldn't settle on a single value. The uncertainty in the line of best fit was also large, with these errors as a large contributing factor.

B. B. Pulsed Laser

The graph showing our experimentally determined speed of light using a pulsed laser is given in Table I.

Test	$S_2(m) \pm 0.02m$	$S_3(m) \pm 0.02m$	$S_4(m) \pm 0.02m$	$S_5(m) \pm 0.002m$	$Time(s)$	Speed(ms^{-1})
1	9.56	11.95	11.8	0.092	222×10^{-9}	$(2.99 \pm 0.0045) \times 10^8$
2	9.56	11.95	7.72	0.092	196×10^{-9}	$(2.98 \pm 0.0045) \times 10^8$
3	9.56	11.95	3.35	0.092	165×10^{-9}	$(3.00 \pm 0.0045) \times 10^8$

TABLE I: Calculated values of light using a pulsed laser

The speed in each case was calculated using the following equation.

$$v = \frac{2(S_2 + S_3 + S_4 - S_5)}{t} \tag{9}$$

Each measurement of distance has an estimated uncertainty of ± 0.02m. This value was decided on because although the tape measure used has a precision of 0.001m it wasn't rigid so stretching and slack had to be taken into account. The measurement of time has an uncertainty of $\pm 0.05 \times 10^{-9} s$, this is half of the smallest increment on the scale used. The uncertainties for the speed values were calculated according to the following method.

$$\Delta S = \pm\sqrt{6(0.02^2) + 2(0.002^2)} = \pm 0.049m$$
$$\Delta t = \pm 0.05 \times 10^{-9} s$$
$$\% e_S = \pm 100 \times \frac{\Delta S}{\Sigma S} = \pm 100 \times \frac{0.049}{9.56+11.95+11.8-0.092} = \pm 0.148\%$$
$$\% e_t = \pm 100 \times \frac{\Delta t}{t} = \pm 0.0225\%$$
$$\% e_1 = \sqrt{0.148^2 + 0.0225^2} = \pm 0.150\%$$

The average value for the speed of light was calculated by weighting the results inversely as a square of the associated error. The calculation is given below

$$\bar{c} = \frac{\Sigma \frac{c_n}{e_n^2}}{\Sigma \frac{1}{e_n^2}}$$

$$\bar{c} = \frac{\frac{2.99}{0.0045^2} + \frac{2.98}{0.0045^2} + \frac{3.00}{0.0045^2}}{3 \times \frac{1}{0.0045^2}} = (2.99 \pm 0.0045) \times 10^8 ms^{-1}$$

This value is consistent with the accepted value for the speed of light

VI. CONCLUSION

In this experiment we determined the speed of light using two methods; a variation on the Michelson-Foucault technique and using a pulsed laser. The determined values were $c = (2.96 \pm 0.05) \times 10^8 ms^{-1}$ and $c = (2.99 \pm 0.0045) \times 10^8 ms^{-1}$ respectively. Both these values are within uncertainty range of the accepted value of the speed of light ($c = 2.99 \times 10^8 ms^{-1}$) and are thus consistent with the theory provided.

The uncertainties in this experiment were due mostly to instrumental errors. The tape measure used for distance measurements was non ideal and a laser range finder would have been more accurate, reducing the uncertainty from a relatively large estimated value to a far smaller calculated value. The method for recording frequency had a propensity for error as it relied on a light beam being incident on the detector with a great enough intensity to generate an accurate measurement. We found that the readings lagged and fluctuated even when the voltage applied to the motor was held constant. With these changes applied the experiment could achieve both a greater precision and accuracy.

VII. REFERENCES

Hecht E. Optics Fourth Edition. Sansome St., San Francisco (CA): Addison Wesley; 2002. 698 p.

Appendices

Appendix A: Raw Data

$Frequency_{read}$(Hz) ±2Hz	Displacement(cm) ±0.0005cm
100	4.3189
198	4.3631
304	4.4015
401	4.4432
503	4.4829
596	4.5348
701	4.5801
803	4.6165
901	4.6844
1001	4.6844

TABLE II: Michelson-Foucault variation experimental data, Frequency ranging from ≈100-1000Hz in 100Hz increments with recorded image displacement.

$Frequency_{read}$(Hz) ±2Hz	Displacement(cm) ±0.0005cm
144	4.2492
255	4.2995
352	4.3472
444	4.3750
549	4.4419
649	4.5300
749	4.4938
850	4.5238
948	4.5609
1053	4.5977

TABLE III: Michelson-Foucault variation experimental data, Frequency ranging from ≈150-1050Hz in 100Hz increments with recorded image displacement.