EXPERIMENT NO. 4

Thermal Radiation: the Stefan-Boltzmann Law

References: Physics for Scientists and Engineers, Serway and Jewett. Sections 40.1
An Introduction to Thermal Physics, Schroeder, Section 7.4

All objects in the universe exchange energy by emitting and absorbing radiation, but some are more efficient at radiating and absorbing energy than others. The universe, as a whole, is the ultimate radiator and emits at a temperature, T, that is currently 2.72548 ± 0.00057 K. This temperature has been decreasing since the epoch ~ 380,000 years after the Big Bang. Understanding the mechanisms involved in the creation and generation of thermal radiation has been one of the fundamental interests of physics and was one of the primary drivers in the early development of quantum mechanics. The key problem involved understanding the spectral distribution of thermal radiation and how this distribution depends on the T of the emitter and the properties of the emitting surface. Classically, the correct wavelength dependence of spectral irradiance could be predicted at wavelengths in the infrared region, but extrapolation of these predictions to shorter wavelengths resulted in what became known as the "ultraviolet catastrophe". Classical theory predicts that the power emitted from heated objects should continue to increase at shorter wavelength, while observations show quite clearly that emission actually decreases at short wavelength. Resolution of this problem came with the work of Max Planck and the exposition of the "Planck distribution function". The analysis that led to this improvement in understanding is based on the concept of quantization, specifically the quantization of energy levels for electrons in solids. It also resulted in the concept of a "blackbody radiator", an object that is 100% efficient in absorbing and emitting radiation at all wavelengths. The universe is the only perfect blackbody, (BB), but BB properties can be achieved over a limited wavelength range for other objects by tailoring composition and geometry. You will make measurements of the radiative properties of a number of real objects in this laboratory experiment and will find how the physical characteristics of the emitter and absorber influence these properties.

Theory

Planck's theory leads to the following distribution function for the wavelength dependence of emission of thermal radiation by a BB at temperature, T

$$I(\lambda, T) = (2\pi hc^2 / \lambda^5) / [e^{(hc/\lambda kT)} - 1] \qquad (1)$$

where $h = 6.626 \times 10^{-34}$ J sec is Planck's constant, k = 1.38 x 10-23 J /K is the Boltzmann constant, and $I(\lambda, T)$ is the spectral irradiance. It's units are W /m^2 /m. Integration of this function over all wavelengths gives the total power emitted per unit area by a BB at temperature T

$$P = \sigma T^4 \qquad \text{watts / m}^2 \qquad (2)$$

where $\sigma = 5.67\text{x}10^{-8}$ watt / $m^2 K^4$ is the Stefan-Boltzmann constant. Equation 2 is the Stefan-Boltzmann Law.

Equation 2 can be easily modified to accommodate emission from a grey (ie. a non-BB) object by introducing an emissivity, ε, where $0 < \varepsilon < 1$. Then

$$P = \varepsilon\sigma T^4 \qquad \text{watts / m}^2 \qquad (3)$$

Radiative transfer of heat between two objects occurs when they are not at the same temperature. The heat transferred into a specific object can be either positive or negative, depending on the temperature difference and the emissivity, ε, of each system.

For an object at T_a with ε_a located inside a chamber at T_b having walls with ε_b, one has

$$\Delta P = \sigma A(\varepsilon_a T_{\mathbf{a}}^4 - \varepsilon_b T_b^4) \qquad \text{watts} \qquad (4)$$

where ΔP is the net power transferred to the object at temperature, T_a with area A. Note that ΔP can be either positive or negative.

Experiment 1: Radiation rates from different surfaces

General information: In this experiment you will use the radiation sensor to directly measure the power emitted by a variety of surfaces at specific temperatures. The experimental configuration is shown in Figure 1.

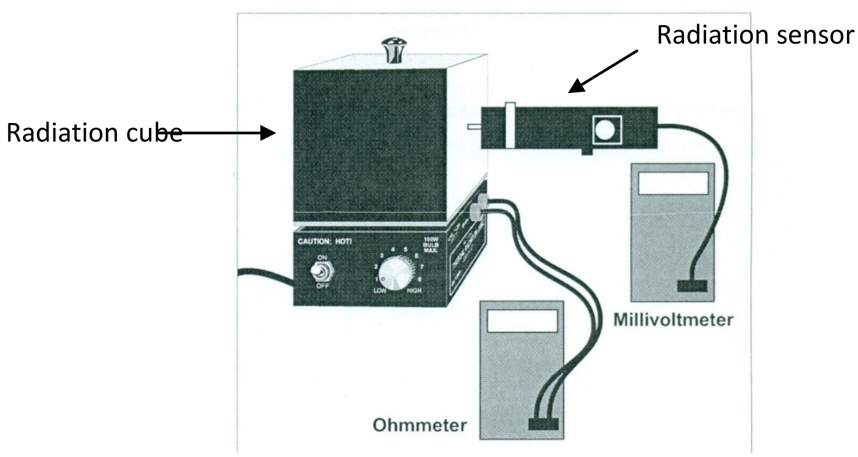

Figure 1. Equipment set up to measure thermal emission from various surfaces. Use a digital multimeter (DMM) as the millivoltmeter and ohmmeter.

The radiation sensor measures the relative intensities of incident thermal radiation and the sensing element, a miniature thermopile, produces a voltage proportional to the intensity of this radiation. The spectral response of the thermopile is essentially uniform in the infrared and visible regions (from 0.5 to 40 μm), and the output voltage generated ranges from the microvolt range up to round 100 millivolts. The sensor is mounted on its stand for more accurate positioning. A spring-clip shutter is opened and closed by sliding the shutter ring forward or back. During experiments, the shutter should be closed when measurements are not actively being taken. This helps reduce temperature shifts in the thermopile reference junction which can cause the sensor response to drift. The two posts extending from the front end of the sensor protect the thermopile and also provide a reference for positioning the sensor a repeatable distance from the radiation cube.

When opening and closing the shutter, it is possible you may inadvertently change the sensor position. Therefore, for experiments in which the sensor position is critical, two small sheets of opaque insulating foam have been provided. Place this heat shield in front of the sensor when measurements are not actively being taken.

The radiation (also called Leslie) cube (Figure 1) provides four different types of radiating surfaces that can be heated from room temperature to approximately 120 °C. The cube is heated by a 100 watt light bulb. You will find 10 individual units available in the laboratory with temperatures ranging from room temperature to ~ 120 °C. Measure the cube temperature for each unit by plugging your ohmmeter into the banana plug connectors labeled THERMISTOR. The thermistor is embedded in one corner of the cube. Measure the resistance, then use Table 1, below, to translate the resistance reading into a temperature measurement. An abbreviated version of this table is printed on the base of the radiation cube.

Part 1. Emission from different surfaces

* Connect the ohmmeter and milli-voltmeter as shown in Figure 1.
* Now use the radiation sensor to measure the radiation signal emitted from each of the four surfaces of each cube. Place the sensor so that the posts on its end are in contact with the cube surface as this ensures that the distance of the measurement is the same for all surfaces. Do not leave the sensor in contact with a surface any longer than required for the measurement as this causes the detector to heat up. Record your measurements in a table. Also measure and record the resistance of the thermistor and use the table 1(page 8) to determine the corresponding temperature. Record your results in tabular form as follows:

Cube number	__1, 2, etc____
Thermistor resistance	______
Temperature	______ °K
Surface	**Sensor reading (mV)**
Black	
White	
Polished Aluminum	
Dull aluminum	

* Based on your measurements of the power emitted by the four different surfaces at various temperatures, and assuming that the black surface has $\varepsilon = 1$ at all temperatures, find ε (T) for each of the other surfaces measured. Is ε for these surfaces a function of temperature? Provide a possible explanation for your findings.

Part 2. Absorption and transmission of thermal radiation

* In turn, place the sensor approximately 5 cm from each surface of the radiation cube having the highest temperature and record the reading, I_0. In each case, then place a piece of window glass between the sensor and the cube and record the new signal, I_g. Find the ratio I_g / I_0 of the two signals. Does window glass effectively block thermal radiation? Does I_g / I_0 depend on I_0? If so, what would account for such a dependence?
* Repeat this measurement for each surface of another cube whose temperature is lower than that in the previous experiment.

Part 3. The Stefan Boltzmann law at low temperatures

In Part 1, you investigated the emissivity aspect of Stefan-Boltzmann Law ($P_{rad} = \varepsilon\sigma T^4$) assuming that the ambient temperature is low enough that it can be neglected in the analysis. In this experiment you will see if this assumption is correct.
If the detector in the radiation sensor were operating at 0 K, it would produce a voltage directly proportional to the intensity of the radiation that strikes it. However, because the sensor is at room temperature, it is also acting as a source of thermal radiation and radiates at the rate $P_{det} = \varepsilon_{det}\sigma T_{det}^4$. The net voltage produced by the sensor is then proportional to incident radiative power minus the power lost by thermal radiation from the sensor.

$$P_{net} = P_{rad} - P_{det} = \varepsilon\, \sigma\, T^4 - \varepsilon_{det}\sigma T_{det}^4 \qquad (5)$$

As long as you are careful to shield the radiation sensor from the radiation cube when measurements are not being taken, T_{det} will be very close to room temperature T_{rm}.

Procedure
Using the data you recorded previously for emission from the black surface of cubes at different temperatures, T_C,

* Determine T_K, the corresponding temperature in °K, for each T_C. Record both sets of data in tabular form. In the same manner, determine the room temperature, T_{rm} (from the room temperature cube)

* Plot a graph of P_{rad} versus $(T_K^4 - T_{rm}^4)$ to verify the Stefan-Boltzmann relation (equation 5) assuming that $\varepsilon = \varepsilon_{det}$.

Experiment 2: The inverse square law

Set up the equipment as shown in Figure 2.

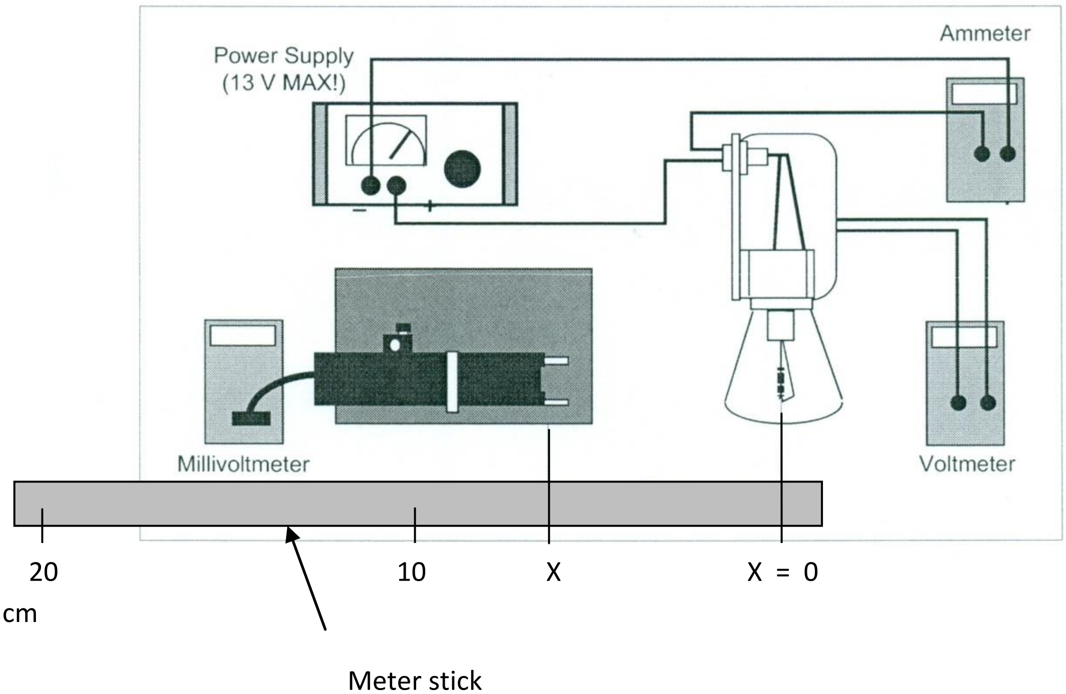

Figure 2

a. Tape a meter stick to the table.
b. Place the Stefan-Boltzmann lamp at one end of the meter stick as shown. The zero-point of the meter stick (X = 0), should align with the center of the lamp filament.
c. Adjust the height of the radiation sensor so it is at the same level as the filament of the Stefan-Boltzmann Lamp.
d. Align the lamp and sensor so that, as you slide the sensor along the meter stick, the axis of the lamp aligns as closely as possible with the axis of the sensor.
e. Connect the sensor to the milli-voltmeter and the lamp to the power supply as indicated in the figure.
* With the lamp OFF, slide the sensor along the meter stick. Record the reading of the millivolt-meter at X = 10, 20, 30... cm. Record your values in tabular form (X, $mV_{background}$). Average these values to determine the ambient level of thermal radiation. You will need to subtract this average ambient value, $mV_{ambient}$, from your measurements with the lamp on, in order to determine the contribution from the lamp alone.

* Put a reflecting/insulating sheet in front of the sensor. Turn on the power supply to illuminate the lamp. Set the voltage to approximately 10 V.

IMPORTANT: Do not let the voltage to the lamp exceed 13 V.

* Adjust the distance between the sensor and the lamp to give a useful number of readings between $X = 2.5$ cm and $X = 100$ cm. To obtain a good plot when the signal is changing rapidly, you will probably need to have more measurements for $X < 20$ cm than at larger distances. At each setting, record the reading, mV_X, from the milli-voltmeter. Make each reading quickly, but let the detector signal stabilize before recording data. Between readings, move the sensor away from the lamp, or place the reflective heat shield between the lamp and the sensor, so that the temperature of the sensor does not rise above room temperature.
* For each value of X, calculate X^{-2}. Enter your results in a table $[X^{-2}, (mV_X - mV_{ambient})]$.
* Plot a graph of radiative power as given by $(mV_X - mV_{ambient})$ versus X^{-2} and another given by $(mV_X - mV_{ambient})$ versus X^{-1}.
* The radiative power from the lamp should scale as X^{-2}, but this is not always observed. Why is there a deviation from the ideal result? Suggest a way in which the experiment could be changed so as to yield more precise measurement of the X^{-2} dependence.

Experiment 3. The Stefan-Boltzmann law

In this experiment, you will determine the temperature dependence of the power emitted from a quasi-blackbody source (a heated tungsten filament) for comparison with that predicted from the Stefan-Boltzmann law. The lamp will be at a relatively high temperature (~1000 K) so the ambient temperature can be ignored, and the calculation of the filament temperature is straightforward.

BEFORE TURNING ON THE LAMP, measure and record T_{ref} , the room temperature in degrees Kelvin, (K = °C + 273) and R_{ref} , the resistance of the filament of the Stefan-Boltzmann lamp at room temperature. **Accuracy is important here**, as a small error in R_{ref} will result in a large error in your result for the filament temperature.
* Set up the equipment as shown in Figure 2. The voltmeter should be connected directly to the binding posts of the Stefan-Boltzmann lamp. The sensor should be at the same height as the filament, with the front face of the sensor approximately 6 cm away from the filament.
*No other warm objects (**including you**) that can be seen by the sensor should be close to the lamp.*
* Turn on the power supply and set the voltage, V, sequentially to V = 1, 2, 3,....12 volts. At each voltage setting, record I, the ammeter reading, and P_{rad}, the reading on the millivoltmeter. Make each sensor reading quickly. Between readings, place both sheets of insulating foam

between the lamp and the sensor, with the silvered surface facing the lamp, so that the temperature of the sensor stays relatively constant.
* Calculate and record R_T, the resistance of the filament at each of the voltage settings used ($R_T = V/I$). Determine the temperature of the filament as follows:

a) For each setting, measure the voltage and current into the filament and divide the voltage by the current to find the resistance R_T.
b) Divide R_T by R_{ref} to obtain the relative resistance (R_T/R_{ref}).
c) Using your measured value for the relative resistivity of the filament at temperature T, use Table 2 on the following page, or the associated graph, to determine the temperature of the filament.

* Calculate T^4 for each value of T and record (P_{rad}, T^4) in a table.
* Plot a graph of P_{rad} versus T and another of P_{rad} versus T^4.
* Find the best fit to the P_{rad} versus T^4 plot and the equation for the regression line together with the standard deviation.
* Estimate the error involved in each step of the calculation of R_T and in the measurement of P_{rad}. Outline how these errors combine to affect the overall quality of the data. Which of these is dominant when T is at the low end of the range used?
* Plot a graph of P_{rad} versus $T_K^4 - T_{rm}^4$. Use P_{rad} as the dependent variable (y-axis) and compare your result to that predicted from the Stefan-Boltzmann relation.

Table 1

Resistance versus Temperature for the Thermal Radiation Cube

Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

Table 2 Temperature and Resistivity for Tungsten

R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

Temperature versus Resistivity for Tungsten