EXPERIMENT NO. 2

Electrostatic and Magnetic Deflection of Electrons in a Cathode Ray Tube

Part A. Motion of electrons in an electric field:

<u>Introduction</u>

The heart of an oscilloscope is the cathode-ray tube (CRT). The electrode structure (electron gun) of a CRT is shown schematically in Fig. 1.

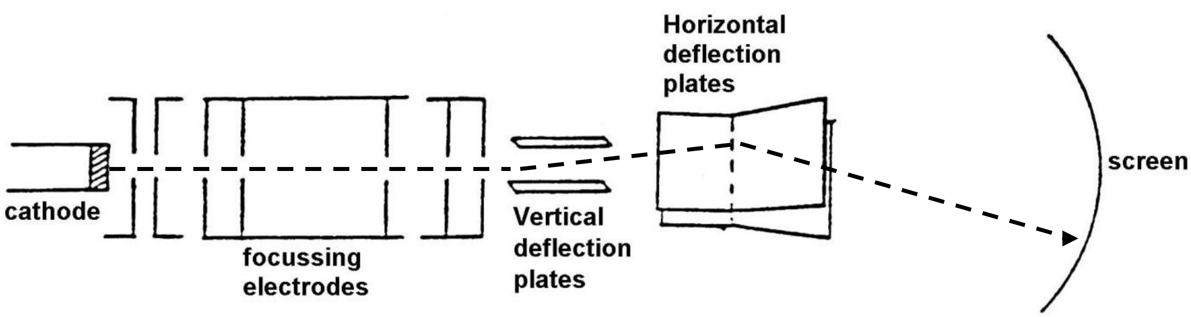


Figure 1: Electrode structure of the CRT

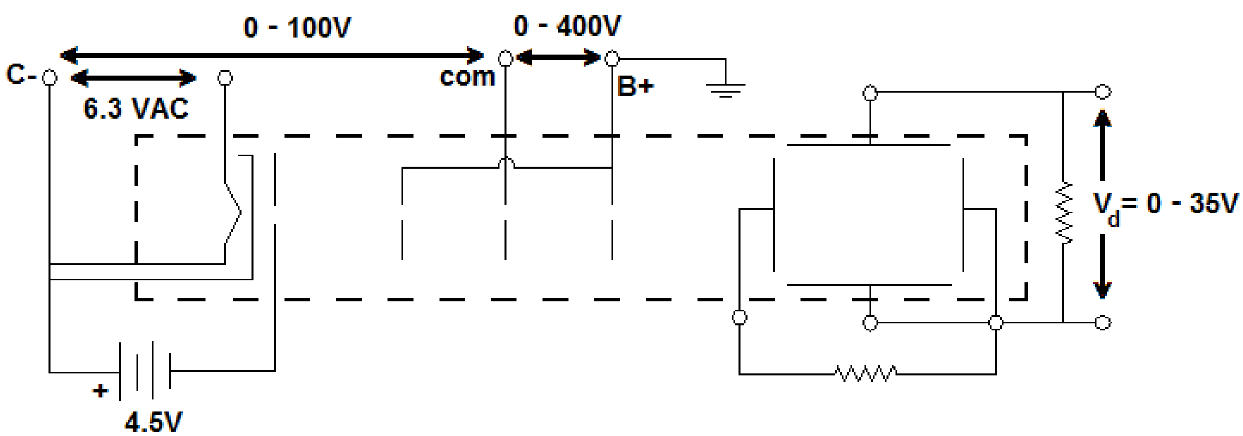


Figure 2: The electrical connections of the CRT

Electrons are emitted from the cathode and are accelerated and focused onto a screen by an assembly of electrodes. Some of the energy of these electrons is converted into visible light when they hit the fluorescent screen, producing a small bright spot. The spot can be moved around on the face of the screen by applying voltages to the deflecting plates.

If the electrons accelerated by the gun fall through a total potential difference V_A, they will have a velocity component, v_z, along the axis of the gun (the z direction) given by the energy equation

$$\frac{1}{2}m{v_z}^2 = eV_A \qquad (1)$$

Ideally, the electrons will have zero velocity in the x and y directions perpendicular to the z axis on leaving the gun, but the earth's magnetic field may give the beam some deflection. Electrons can be accelerated transversely by applying potential differences between the deflecting plates. Let us consider one set of plates only, those that can cause vertical deflections.

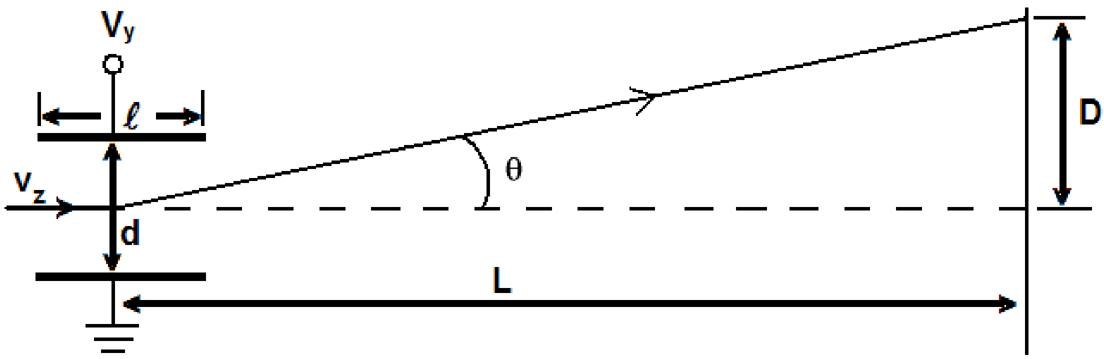


Figure 3: Electron deflection in a CRT.

On the <u>assumption</u> that the electrostatic field is uniform between the plates and extends only over the distance ℓ, the field between these plates is the applied voltage divided by the plate separation. Therefore:

$$E_y = V_y / d, \qquad (2)$$

and an electron will be in this field for a time

$$t = \ell / v_z \qquad (3)$$

The force on the electron during this time interval is $F_y = e \cdot E_y$ and the vertical component of acceleration will be given by

$$a_y = \frac{F_y}{m} = \frac{e \cdot E_y}{m} = \frac{e \cdot V_y}{d \cdot m} \qquad (4)$$

This can be used to obtain the change of velocity in the vertical direction,

$$v_y = a_y \cdot t = \frac{e}{m}\frac{V_y}{d}\frac{\ell}{v_z} \qquad (5)$$

The tangent of the deflection angle, θ, is given by the ratio of the velocity components,

$$\tan\theta = \frac{v_y}{v_z} = \frac{eV_y\ell}{mdv_z^{\,2}} = \frac{\ell}{2d}\frac{V_y}{V_A} \tag{6}$$

If D is the deflection of the beam on the screen, and L is the distance from the centre of the deflection plates, making the approximation L >> ℓ , Fig. 3 shows that

$$D \approx L \tan\theta \approx \frac{L\ell}{2d}\frac{V_y}{V_A} \tag{7}$$

The dependence of D on V_y and V_A

The lab instructor will acquaint you with the apparatus, instruments and their operation. After this is done, start with the case of a high accelerating potential by setting the V_{C-} adjustment of the large power supply to its maximum output position (100%) and then adjust the V_{B+} dial to obtain a well focussed spot on the CRT tube face. Measure your CRT's accelerating potential V_A (=$|V_{B+}|+|V_{C-}|$) using a multi-meter. You may find it convenient to rotate the CRT on the desktop to obtain an orientation for which the spot lies on the axis when the deflecting plate voltage is absent.

* What do you suspect is the origin of this spot motion when the tube is rotated?

*Answer all questions beginning with * in your report as well as other questions in the text.*

Once this orientation is achieved, the CRT must be kept in this position for the rest of this part of the experiment.

With your small power supply connected to the vertical deflection plate leads and your voltmeter on the appropriate range, deflect the spot vertically in increments of the major vertical divisions and note the voltage V_y, required to do this. Your deflections should span the entire vertical scale, which will require you to reverse the polarity of the plate voltage in order to deflect the spot in the opposite direction. This is achieved by simply removing, reversing and re-inserting the leads plug to the V_y source. The major divisions of the graticule scale on the tube face are ¼ inch units (=0.635 cm) and must be converted to centimetres for your analysis. Repeat this procedure for a lower V_A value by setting V_{C-} to approximately 50% of its maximum output and again adjusting V_{B+} to obtain a well-focussed spot.

Tabulate all the data obtained such that upward (or +D) deflections are associated with $+V_y$ voltages and downward (or –D) deflections are associated with $-V_y$ voltages. Plot D against V_y for both values of V_A on one graph.

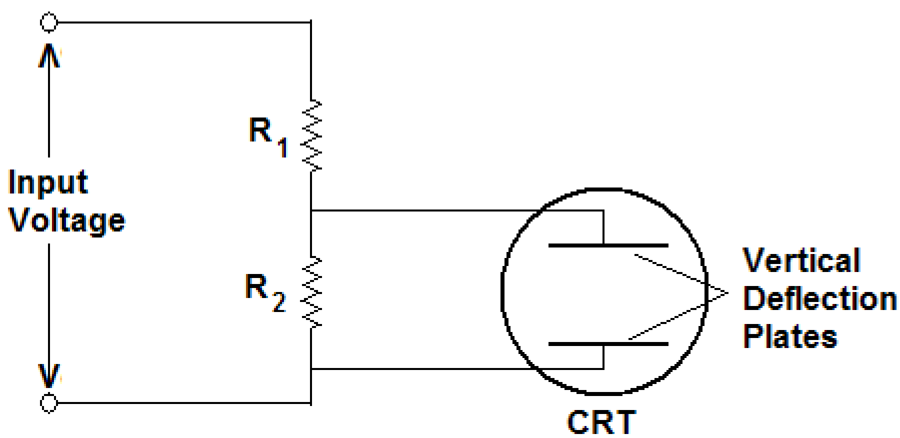
*Are your graphical results as expected?

Calculate V_y/V_A for both sets of V_A values and plot D against V_y/V_A on a second graph.

*Are your graphical results in the second plot as expected?

Determine the Deflection Coefficient, DC, for the tube where DC = $\Delta V_y/\Delta D$ (Volts/ugd*). The deflection coefficient tells you the required input voltage to vertical deflection plates necessary to produce a unit vertical displacement of the spot on the tube face.

Equation (7) provides the basis for understanding the oscilloscope as an instrument which will measure voltage, time and hence, frequency and phase. To emphasize the close connection between the CRT itself and the oscilloscope, consider the following diagram of the input connection to a CRT. The resistor R_2 is a "bleed resistance" across the vertical deflection plates. Its purpose is to remove charge from the plates to neutralize them. Your input wires to the deflection plates have such a resistor.



<u>Creation of an Adjustable Calibrated Voltage Scale</u>

a) Find an expression for the Deflection Coefficient for the tube, DC, in terms of the slope of graph D vs. V_y/V_A.

b) What V_A must your CRT have to achieve a Deflection Coefficient, DC, equal to 1 volt/ugd? [ugd = unit graticule division]

c) Assuming you have a DC for your tube equal to 1 volt/ugd, what values of R_1 are required to achieve an effective deflection coefficient, $DC_{eff} = \frac{\Delta(Input\ Voltage)}{\Delta D}$ (as shown in the figure above) of 1, 2, 5 and 10 volts/ugd? Give details.

d) Why is it not possible to obtain all these values with your CRT?

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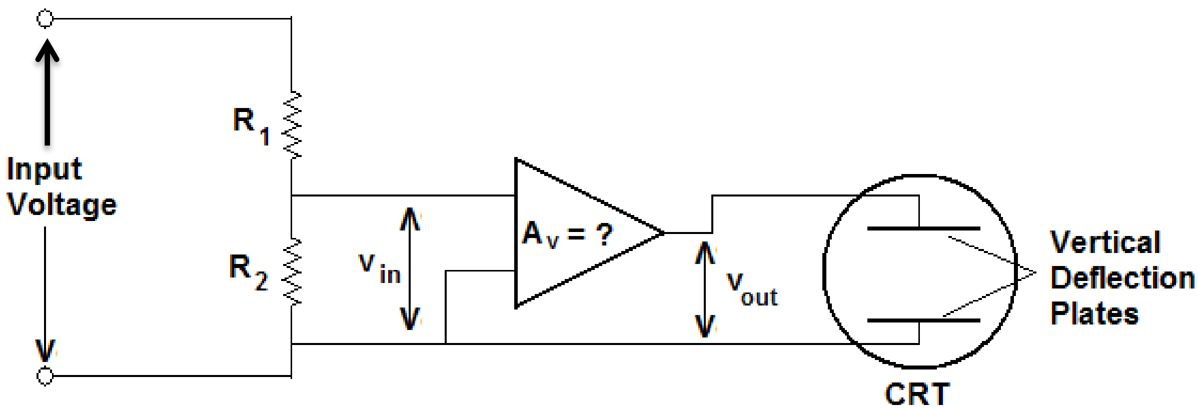
* ugd = unit graticule division

e) The correction for the problem of part (d) is to raise the accelerating potential across the electron gun, V_A. Assuming the answer to (b) is 40 volts, let's assume we set the V_A to 400 volts. What is the new DC as a result of these changes? If 1 volt is now placed across R_2 what spot displacement results? What voltage must be placed across the plates to now achieve a spot displacement of one ugd?

f) The solution to the higher V_A value which retains the resistor divider values computed in (c) is to have an amplifier with voltage gain, $A_V = v_{out}/v_{in}$ inserted between the output voltage across R_2 and the input voltage to the deflection plates as shown below. What is the required value of A_V?



The relationship which permits one to measure the input voltages to such a system should be seen to be:

$$V_i = DC \cdot D_y$$

where V_i is the input voltage across R_1 and R_2 and D_y is the deflection of the spot in ugd's. This equation indicates that the vertical (or horizontal) size of the trace is controllable by the magnitude of the DC of the system and/or that of the input voltage, V_i.

Creation of an Adjustable Calibrated Time Base

A time base, like that possessed by an oscilloscope in the x-axis direction, is based on the achievement of an adjustable constant speed of the beam spot in the direction from left to right across the CRT tube face. An electronic circuit is designed to achieve such a result.

To understand how this can be accomplished, consider the DC expression in the form below where ΔD and ΔV_x are divided by the common time interval Δt. V_x implies that the voltage is to be placed across the horizontal deflection plates. Recognizing that $\Delta D / \Delta t$ equals the speed of the spot in the x-direction, v_x,

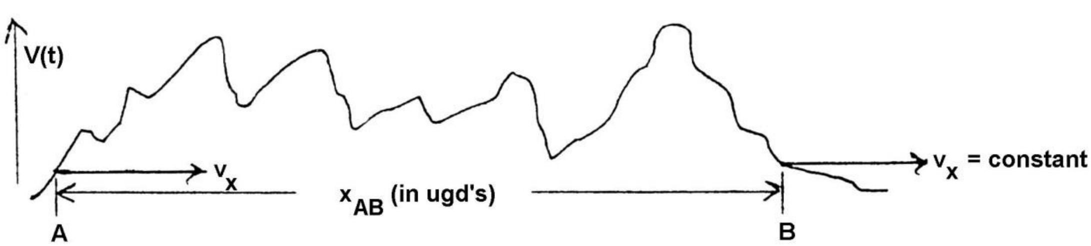
$$\Delta D = \frac{1}{DC}\Delta V_x \rightarrow v_x = \frac{\Delta D}{\Delta t} = \frac{1}{DC} \bullet \frac{\Delta V_x}{\Delta t}$$

a) Assuming that V_A is 400 volts and that the DC for the horizontal deflections plates is the same, how must V_x vary with time in order to achieve a constant speed? What must $\Delta V_x/\Delta t$ be in order for v_x to be 1 ugd/sec? What is the Time Equivalent/ugd, TC, under this circumstance for the motion of the spot in the x-direction? If two points on a trace of a displayed voltage, $V_y(t)$, are measured to be 4.6 ugd's apart, what is the time interval between these two points?

b) What magnitudes of $\Delta V_x / \Delta t$ are required to achieve a time base, TC, of 2 sec/ugd and 0.2 sec/ugd?

From the diagram below it is evident that time base for the horizontal axis relates time and distance according to:

$$\text{Time} = (\text{time / ugd})_{cal} \cdot x_{ugd} = TC \cdot x_{ugd}$$



$$T_{AB} = x_{AB}/v_x = (\text{Time/ugd})_{cal.} \cdot x_{AB(ugd)} = TC \cdot x_{AB(ugd)}$$

Hence, in such an oscilloscope, controls on the vertical and horizontal inputs to the deflection plates give a graphical display of V(t) vs. time (y vs. x) where the voltage equivalents per ugd and the time equivalents per ugd come from the calibrated DC and TC settings.

Part B. Motion of electrons in magnetic fields.

References: Physics for Scientists and Engineers, Serway and Jewett. Sections 29.4, 29.5
Elements of Electromagnetics, Sadiku, 3rd/4th editions

Introduction

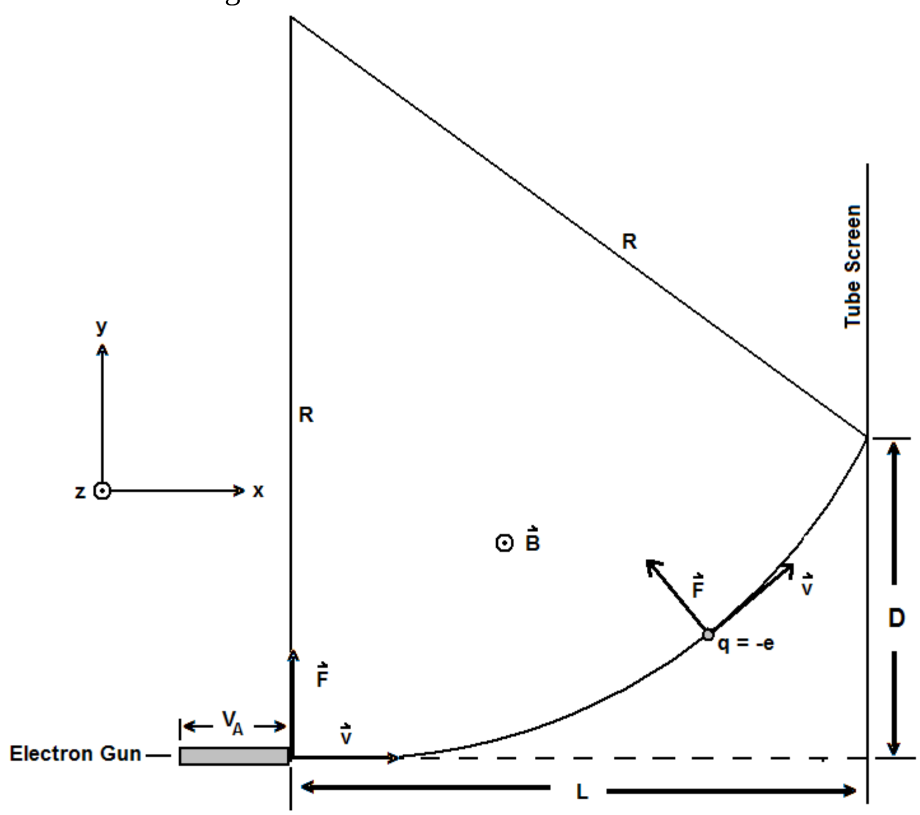
The motion of a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is governed by the dynamical relation

$$\vec{F} = m\frac{d\vec{v}}{dt} = q\vec{E} = q\left(\vec{v}\times\vec{B}\right) \tag{8}$$

In this experiment the electron beam $(q\vec{v})$ of a CRT is allowed to interact with magnetic fields. The resultant motion of these interactions can be analysed by applying equation 8 to produce both qualitative and quantitatively verifiable relationships between measurable parameters. Verification of these relationships in turn serves to establish the validity of equation 8.

Theory

Consider the magnetic deflection of an electron beam where the magnetic field is directed perpendicular to the beam and is uniform throughout the region of the tube. Schematically we have the following:



Charge q exiting the electron gun experiences the following force

$$m\frac{dv}{dt} = q(v \times B)$$

$\vec{v} = v\hat{i}$ and $\vec{B} = B\hat{k}$ where q = − e

therefore $v_y = \frac{e}{m}\int_o^t vBdt$ (9)

(note that the negative charge changes the sign of v_y)

Since v = $\sqrt{v_x^2 + v_y^2}$ and <u>assuming</u> that B is sufficiently weak such that $v_y \ll v_x$ over length L

$$v \simeq v_x$$

dx = v_x dt

and equation 9 becomes

$$v_y \simeq \frac{e}{m}\int_o^x Bdx \qquad (10)$$

The displacement of the spot on the tube face is given by

$$D = \int_o^T v_y dt$$

but dt $\simeq \frac{1}{v_x}dx$

therefore $D = \int_o^L \frac{v_y}{v_x}dx$ (11)

Substituting (11) into (10) gives

$$D \simeq \frac{e}{m}\frac{1}{v_x}\int_o^L\left(\int_o^x Bdx\right)dx \qquad (12)$$

Since we are considering the case where B is uniform over the tube region, equation (12) integrates to

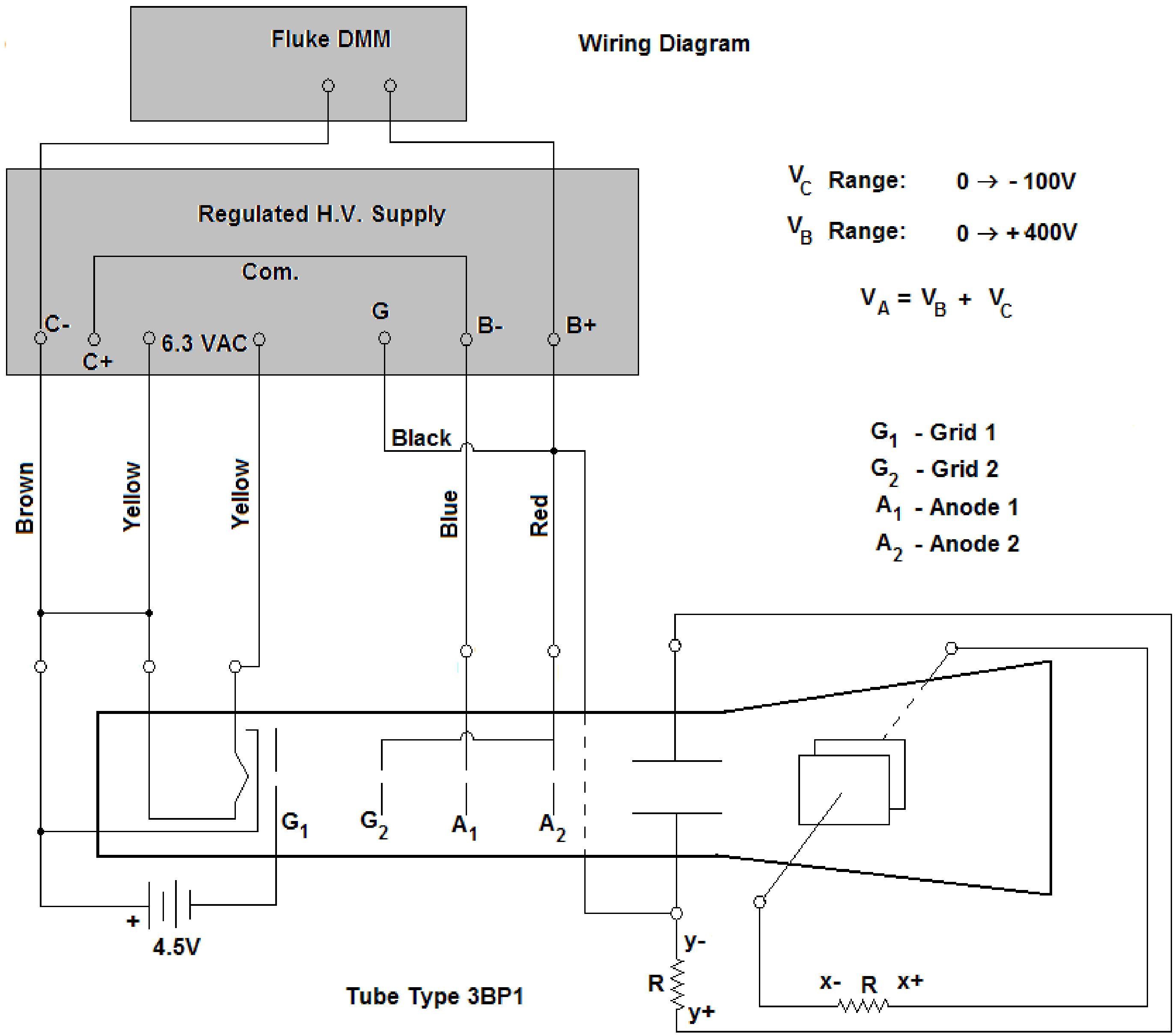
$$\boxed{D \cong \frac{eB}{mv_x}\frac{L^2}{2}} \qquad (13)$$

Apparatus

The wiring of the CRT should be hooked up for you. If it is not, then proceed with the wiring according to the schematic figure on the next page. Under either circumstance, you should **not turn on** any power until your lab instructor has checked the circuit. The B-, C+ is the COM input jack on your regulated power supply and the B+, G and the C-, heater "jumper" wires are already connected for you. The bias battery is located inside of the tube housing. **Great care** should be exercised in keeping your hands away from this battery as it may be set as much as 500 volts below ground.

If not already done for you, connect the x-deflection plate leads of the CRT to the GND terminal of the Regulated HV supply and the y-deflection plate leads to the output of EUW-17 power supply. A wire to the GND terminal of the Regulated HV supply should connect the negative output of the EUW-17 supply. The output of this supply should be set to a maximum so that when it is turned on, (i.e. when you're not using the tube) the spot will be deflected off screen, which will protect the phosphor coating of the tube from being burned. To use the CRT, turn the EUW-17 voltage supply off, and allow the spot position to stabilize on the screen before proceeding. This will ensure that there will be no electrostatic influences from the deflection plates during its use.

The presiding lab instructor will give instructions on the use of the Regulated HV supply and any additional instructions for the other instruments used.



Experiment 1

The determination of the direction and magnitude of the Earth's magnetic induction, B

Adjust the V_C and V_B voltages on the Regulated Power Supply, as instructed to obtain a well-focussed spot for an accelerating potential V_A of approximately 250V. Pivot the CRT on its back edge on the desk and orient the tube on this edge to a position at which there is no spot deflection (i.e. the origin of the graticule). Take care not to disconnect any wires in the process. Use the protractor provided make an accurate measurement of the angle of elevation of the tube (with respect to the desktop) and measure the angle between the projection of the tube axis in the horizontal plane and the direction towards the front of the lab room. These angular measurements will define the direction of the earth's magnetic field, using the room as a reference system.

* What is the direction of the velocity vector $\vec{v}_x$ and the field vector $\vec{B}$ in this situation? Make reference to a pertinent equation in answering this question.

To determine the magnitude of $\vec{B}$ orient the CRT in space to find the maximum linear deflection of the spot along the x (or y) axis on both sides of the graticule origin. Measure the distance between these extreme positions in unit graticule division (ugd) and convert it to centimetres where 1 ugd = 0.635 cm. Using equation 13, (which is exact in this situation) determine the magnitude of $\vec{B}$. The distance L from the second anode A_2 to the screen has been measured for you and is 16.0 ± 0.2 cm.

For the case of maximum deflection, what is the direction of $\vec{v}_x$ with respect to $\vec{B}$? Justify your answer making use of an appropriate vector equation.

By rotating the CRT with its base flat on the desktop determine, in a similar manner, the magnitude of the horizontal component of the earth's field: $\vec{B}_H$. Note and record your observations of the spot position as this is done, and explain them with reference to appropriate equations.

* How does your value for the magnitude of $\vec{B}_H$ compare with the approximate value quoted for this region of 2.1×10^{-5} Wb/m^2?

From your measurements of $\vec{B}_H$ and $\vec{B}$, calculate the angle of elevation $\vec{B}$ makes with the plane of the table top.

* How does this angle compare with the measured angle obtained previously?

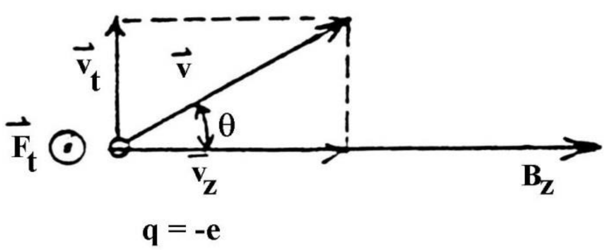
With your measurements for the direction of $\vec{B}$ and your observations of the direction of beam deflection, draw a diagram showing the "actual" direction of $\vec{B}$. Orient the coordinates x,y,z relative to the room.

Finally, determine the maximum absolute error in B, and state the results of your measurements in the form of B ± ΔB.

Experiment 2

Helical motion of electrons and magnetic focussing

The previous exercises have dealt with uniform magnetic fields directed transversely to the direction of the beam. This gave rise to spot deflections resulting from a radially directed central force due to the q, v, B interaction. In this exercise, we will consider the effect on the beam motion when it is given velocity components both parallel with and transverse to the magnetic field. The field will be directed down the axis of the tube and the transverse component of the velocity is created using the electrostatic deflection plates in the CRT. Our analysis of this situation will reveal the capability of the system to magnetically focus the beam.



Consider the axis of the tube as the z-direction and a magnetic field directed as shown. An electron, q = − e, has a velocity v with components v_z and v_t where v_t is perpendicular or transverse to the field B_z. The force acting on the electron under this condition is given by

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right) = q\left(\vec{v}_t\times\vec{B}_z\right) + q\left(\vec{v}_z\times\vec{B}_z\right)$$

In this case, $\vec{F}_z$ is zero because $\vec{v}_z$ and $\vec{B}_z$ are parallel, therefore:

$$\vec{F} = \vec{F}_t + \vec{F}_z = \vec{F}_t \qquad (14)$$

Separating the resultant motion of the beam into two components, that in the z-direction and that in the plane perpendicular to this direction, motion in the z-direction is described by the constant velocity equation

$$z = v_z\, t \qquad (15)$$

since the force (acceleration) in this direction is zero. In the plane perpendicular to the z-direction, the beam executes circular motion determined by a radial central force of magnitude

$F = q\, v_t B_z$.

From mechanics, we can equate this to the centripetal force to give

$$q\, v_t\, B_z = \frac{mv_t^2}{r}$$

$$\therefore\ r = \frac{mv_t}{qB_z} \qquad (16)$$

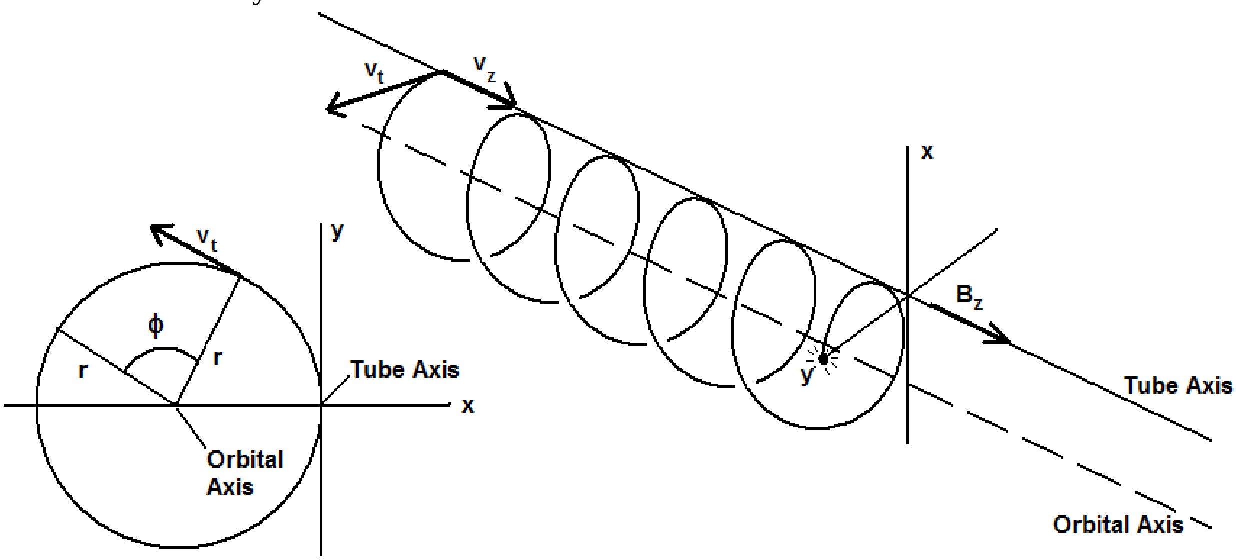
where r is the radius of the circular motion and m is the mass of the electron. Since m v_t /q is constant in this case but B_z can be varied, we expect to observe that r α $1/B_z$. Another fact about its motion is realized if we replace the transverse velocity v_t, which is in fact the tangential velocity of the circular motion, by its relationship to the angular frequency of rotation and the radius of the circular motion given by

$$v_t = \omega\, r \qquad (17)$$

Substituting back into the r expression reveals the relation between ω and B_z

$$\omega = \frac{q}{m} B_z \qquad (18)$$

Since B_z will vary, we expect ω to respond in a directly proportional manner (i.e. ω α B_z). The combination of these two effects cause the beam to execute a helical motion down the tube as shown schematically below.



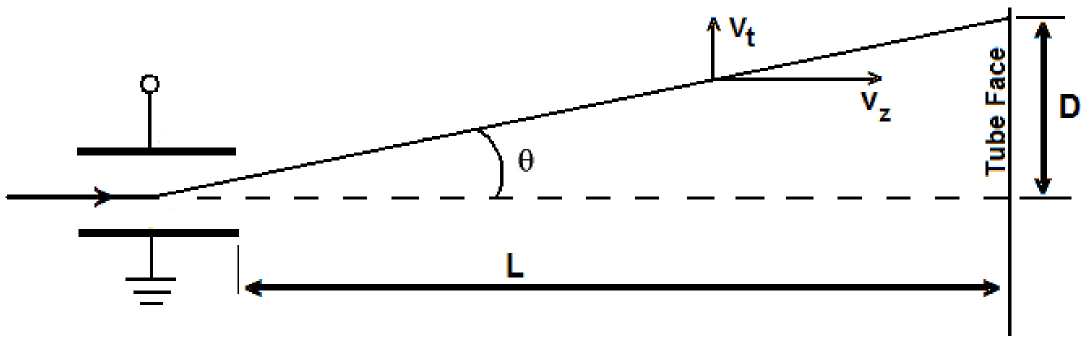
The angle of rotation (azimuthal angle), of the electrons about the orbital axis in time t is given by $\phi = \omega t$. Since the z-direction translational position is given by $z = v_z\, t$, combining these equations gives

$$z = \phi \frac{v_z}{\omega} \tag{19}$$

The pitch, p, of this helical motion is defined as the z-axis distance travelled by the electron in rotating through an angle of 2π radians. Hence setting z = p for $\phi = 2\pi$ gives

$$p = 2\pi \frac{v_z}{\omega} \tag{20}$$

In the set-up used, a transverse velocity component is introduced via a pair of electrostatic deflection plates as shown below.



From the geometry we see that

$$\tan\theta \cong \frac{D}{L} = \frac{v_t}{v_z} \tag{21}$$

Hence, the electrons will move down the tube on a helix of radius

$$r = \frac{v_t}{\omega} = \frac{v_z \tan\theta}{\omega} \tag{22}$$

with a pitch,

$$p = 2\pi \frac{v_z}{\omega}$$

As seen from the diagram the distance from the end of the deflection plates to the screen is z = L, so that,

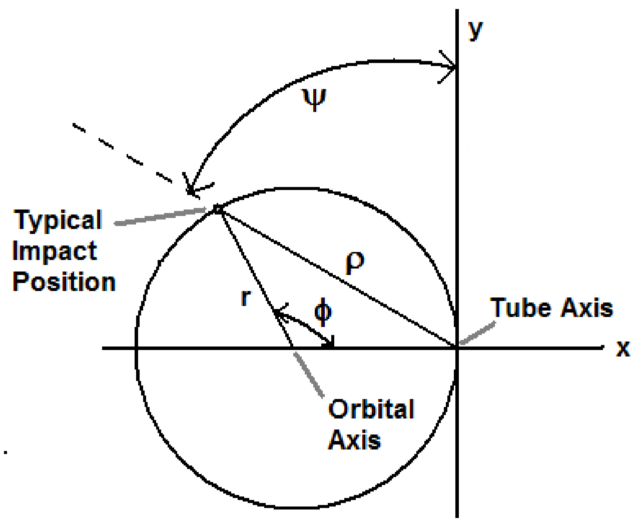
$$L \doteq \phi \frac{v_z}{\omega} \tag{23}$$

which is the z-axis distance traversed by the electron before hitting the CRT face. But we know that $v_z / \omega = p/2\pi$. Therefore, eliminating v_z / ω gives

$$\phi = 2\pi \frac{L}{p} \tag{24}$$

the azimuthal angle of rotation that takes place over the distance L. Note that $\phi \propto 1/p \propto 1/r \propto B_z$.

To understand what is seen on the tube face, remember that the electrons move in a helix the centre of which (the orbital axis) is displaced transversely from the centre of the screen (tube axis) by the radius r. Introducing polar coordinates (ρ, ψ) we can conclude the following relationships describing the spots radial position, ρ, from the tube axis, in terms of the angle ψ as shown.



From the geometry,

$$\rho = 2r\sin\frac{\phi}{2}$$

Using the appropriate substitutions for r from above we have:

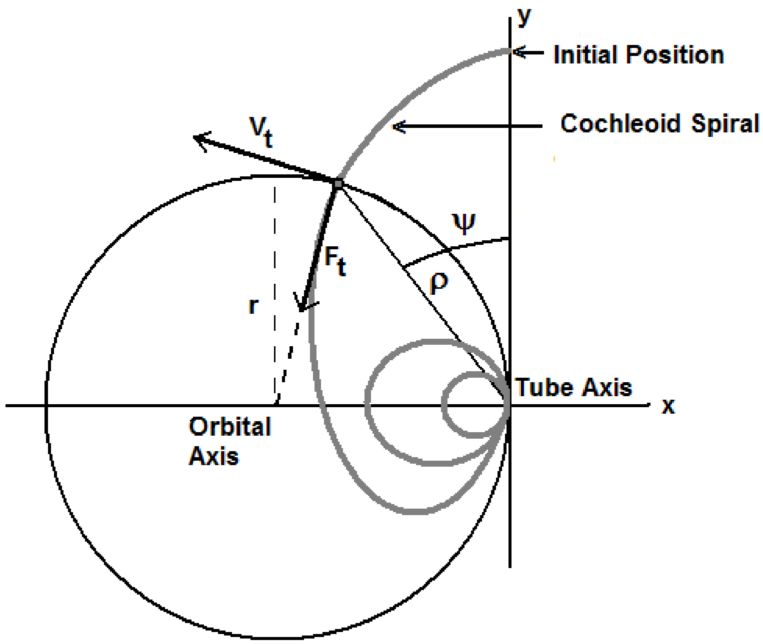
$$\rho = \left(\frac{p}{\pi}\tan\theta\right)\sin\frac{\phi}{2}$$

Now we know, when the electron impacts with the phosphor over length L the angle of rotation is given by $\phi = 2\pi\frac{L}{p}$

From the geometry it is seen that: $\psi = \frac{\phi}{2} = \pi \frac{L}{p}$, hence $\frac{p}{\pi} = \frac{L}{\psi}$ (25)

Upon substitution we see that: $\rho = (L \tan\theta) \frac{\sin\varphi}{\varphi}$ (26)

which is the equation of a Cochleoid spiral and represents the spot motion as seen on the tube face of the CRT.



Now if we realize that equation (26) can be used to describe all the positions of the cross-sectional area of a defocused beam with respect to the common tube axis, and that the ρ displacements of each position go to zero independent of the initial deflection angle θ, then we can see that a defocused beam can be focused magnetically! The focusing condition is obtained by setting

$$\rho = 0 = (L \tan\theta) \frac{\sin\varphi}{\varphi}$$

which occurs for $\sin \varphi = 0$ i.e. for $\varphi = N\pi$ (N an integer)

But $\varphi = \pi \frac{L}{p}$. Therefore, focussing occurs whenever,

$$\frac{L}{p} = N \qquad (N = 1, 2, 3, ...) \qquad (27)$$

Remembering that for phosphor impact,

$$\phi = 2\pi \frac{L}{p} = \omega \frac{L}{v_z}$$

then, for focussing to occur,

$$\omega = \left(\frac{2\pi v_z}{L}\right)\mathrm{N} \tag{28}$$

But, ω = (q/m) B_z, and the N^{th} focus condition would require a z-axis field, B_{zN}, such that

$$\omega = \frac{q}{m}B_{zN} \tag{29}$$

Now, B_{zN} = K i_{SN} = K V_{SN}, where V_{SN} is the voltage across the solenoid giving a solenoid current, i_{SN}, for an axial field B_{zN} necessary for the N^{th} focus to occur.

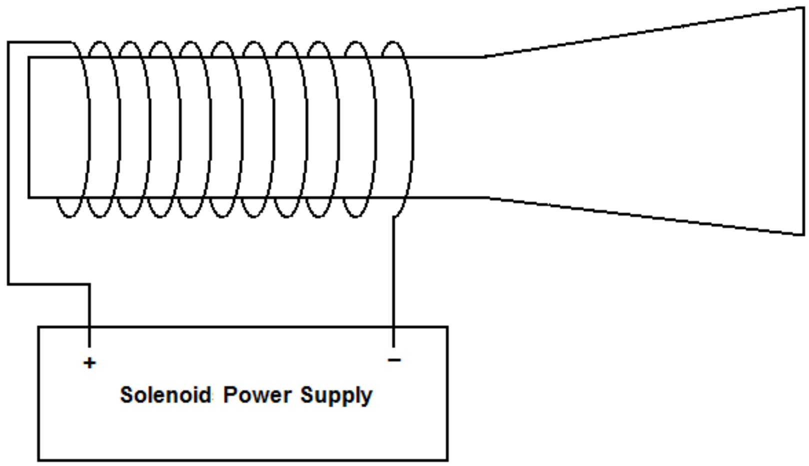
Since, ω = q/m B_{zN} = C V_{SN} then: $\omega = \left(\frac{2\pi v_z}{L}\right)\mathrm{N}$ (30)

Thus we have the following predicted relationship between the solenoid voltage giving the N^{th} focussing situation, V_{SN}, and N , which is

$$V_{SN} = \left(\frac{2\pi v_z}{CL}\right)\mathrm{N}$$

or V_{SN} = [constant] N (31)

To observe the predicted behaviour and verify the above V_{SN}, N relation use the apparatus set up by the presiding instructor at the back of the room. Observe any instructions at the experiment station regarding the operation of the apparatus which is schematically shown below:



Proceed as follows: With the beam focussed and initially deflected off the CRT face, increase the solenoid voltage V_S ($V_S \alpha B_z$). Sketch your observations of the spot motion over the full range of V_S, and then return V_S to zero. Defocus the beam and increase V_S again over its entire range noting the solenoid voltages V_{SN}, for each of the N focus situations you observe. Plot your results to verify the predicted V_{SN}, N relation and comment on any deviations from the expected plot. From your sketch of the spot motion on the tube face, determine the direction of the axial magnetic field.

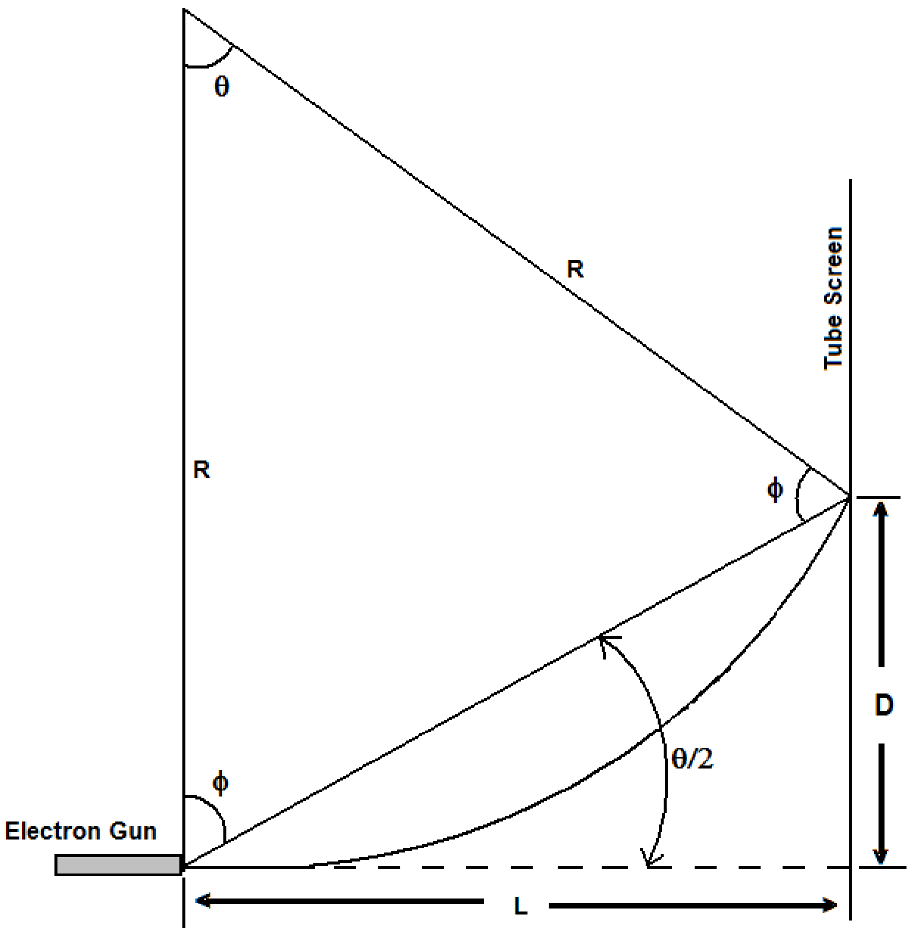
* In general, are the predicted responses of the beam to the increasing axial magnetic field, as reflected in the spot motion on the tube face, realized?

Magnetic focussing is used in modern day electron microscopes because of its superiority to electrostatic focussing. Electron microscopes also give rise to images which are rotated with respect to the sample orientation. This phenomenon is similar to the spot rotation encountered in this experiment if one considers the magnetically focussed electron as a carrier of information.

List of Apparatus

1	Fluke 8050A Digital Multimeter (Voltage accuracy: ± 0.03% of reading + 2 digits)
1	Data Precision Model 1350 Digital Multimeter (Voltage accuracy: ± 0.1% of reading ± 1 l.s.d.)
1	EUW-17 Transistorized Power Supply
1	PMC Regulated Power Supply
1	Heath Kit Regulated H.V. Supply Model 1P-2717A
1	C.R.T. Type 3BP1A
1	Protractor
2	Deflection Solenoids Wires

Appendix:



The deflection, D, of an electron beam over an axial distance L when subjected to a weak uniform magnetic field can be more easily determined using geometry.

$$\frac{L}{R} = \sin\theta \qquad \text{if } \theta \text{ small } \theta = \frac{L}{R}$$

$$\frac{D}{L} = \tan\theta \qquad \text{if } \theta \text{ small } \frac{\theta}{2} = \frac{D}{L}$$

therefore D = $\frac{L^2}{2R} = \left(\frac{1}{R}\right)\frac{L^2}{2} = \frac{qB}{mv}\frac{L^2}{2}$

using the fact that $qvB = \frac{mv^2}{R}$, hence $R = \frac{mv}{qB}$