EXPERIMENT NO. 1

The Measurement of e/m for the Electron

References: Physics for Scientists and Engineers, Serway and Jewett. Sections 29.4, 29.5
Elements of Electromagnetics, Sadiku, 3rd/4th editions

The ratio of the electron's charge to its mass (e/m) was originally measured by J.J. Thompson in 1897. However, this experiment represents a simplified and improved version described by K.T. Bainbridge in 1938.

A specially designed tube filled with mercury (Hg) vapour is mounted at the centre of a pair of Helmholtz coils. The details of the tube design are shown in Figures 1 and 2. A beam of electrons is produced in the tube by an electron gun consisting of a straight filament, F, surrounded by a co-axial anode, C, which contains a single axial slit, S. Electrons emitted from the filament are accelerated due to the voltage applied to the anode. The electrons coming out through the slit form a narrow beam and those electrons with sufficient kinetic energy (> 10.4 electron-volts (eV)) will ionize Hg atoms via inelastic collisions. The resulting ions are rapidly neutralized by recombination with stray electrons leaving Hg atoms in an electronically excited state. As they de-activate, these atoms can emit light at a variety of wavelengths, including lines at 546.1, 435.8 and 404.7 nm, giving the emission a characteristic blue colour. Since this recombination occurs near the site of the original collision, this allows one to "see" the path of the electrons.

A current is passed through the Helmholtz coils, producing a magnetic field that bends the electrons in a circular path. By proper control of the strength of the magnetic field, the sharp outer edge of the beam can be made to coincide with any of the five bars, A, which are set at known distances from the filament.

Figure 1

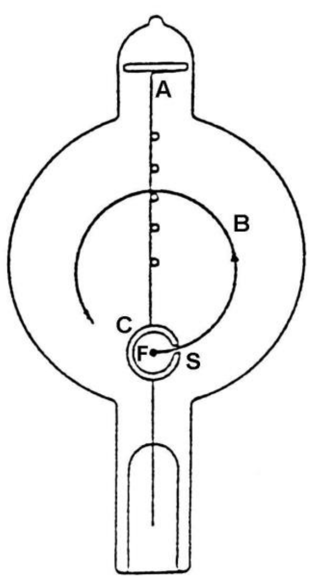
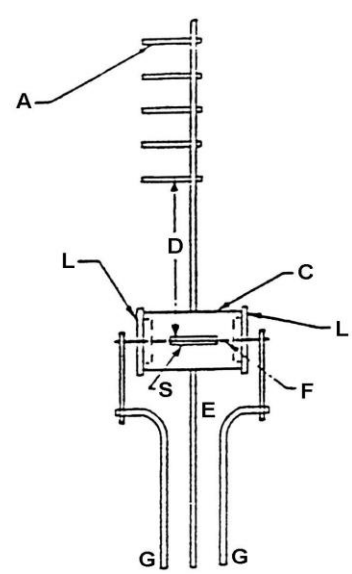


Figure 2



Theory

An electron whose charge is e, accelerated through a voltage V, acquires a velocity, $\vec{v}$, where

$$v = \sqrt{\frac{2eV}{m}}$$

If the electron then moves into another region containing a magnetic field, $\vec{B}$, applied in a direction perpendicular to the velocity of the electron, and where the electric field, $\vec{E}$ = 0, it experiences a force, $\vec{F}$, in a direction given by the cross product of $\vec{v}$ and $\vec{B}$

$$\vec{F} = e\,\vec{v} \times \vec{B}$$

The direction of $\vec{F}$ is then perpendicular to the plane defined by $\vec{v}$ and $\vec{B}$. This force will cause the electron to follow a circular path of radius R, where

$$F = \frac{mv^2}{R}$$

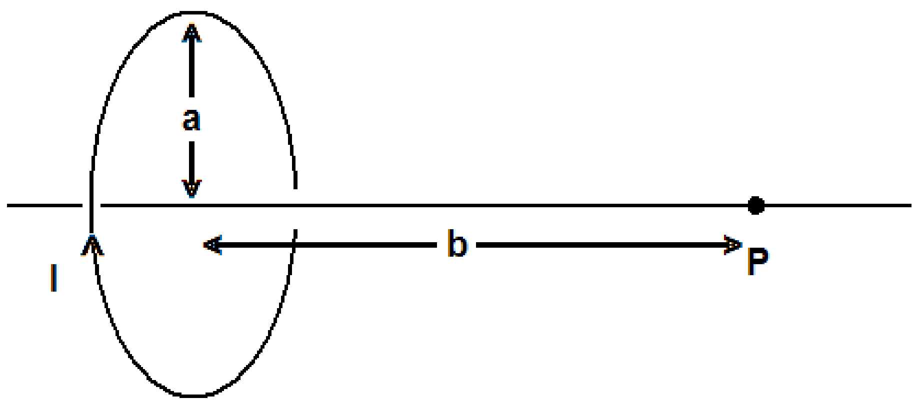
Combining these three equations, D=2R and ignoring the vectors since the directions have been defined, one gets

$$\frac{e}{m} = \frac{8V}{B^2D^2} \qquad (1)$$

where V and D are known from measurement. B can be found from the relationship between the magnetic field and the current in the Helmholtz coils.

The magnetic field, B, at a point on the axis of a single turn of wire carrying current I is:

$$B = \frac{\mu_o I a^2}{2\left(a^2 + b^2\right)^{3/2}}$$



Where a is the radius of the coil and b the distance of the point from the centre of the coil. In SI units $\mu_0 = 4\pi x10^{-7}$ H/m is the magnetic permeability of free space.

The configuration of the Helmholtz coils consists of a pair of coils, arranged as shown below, separated by a distance equal to their common radius. Figure 4 shows a cross-sectional view of two such coils. If current passes in the same sense (clockwise or counter-clockwise) around both coils, then it produces an almost uniform magnetic field B along the central axis and in most of the central region between the coils.

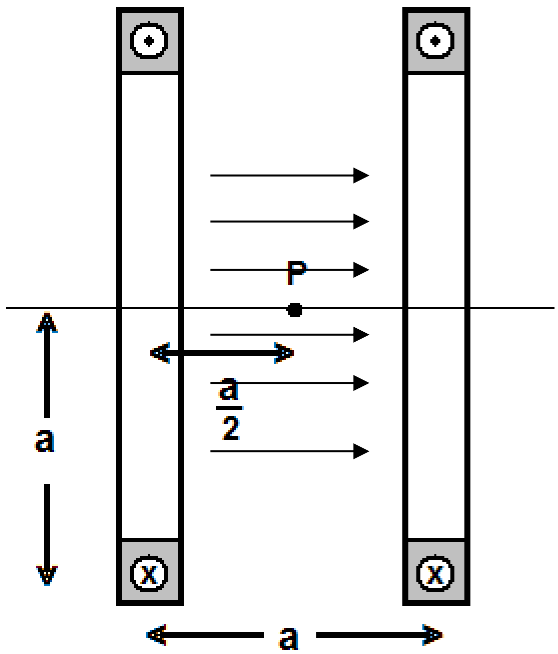


Figure 4

If each coil has N turns, the magnitude of B at the point P, midway between the coils is:

$$B = \frac{\mu_o N I a^2}{2\left[a^2 + (a/2)^2\right]^{3/2}} + \frac{\mu_o N I a^2}{2\left[a^2 + (a/2)^2\right]^{3/2}}$$

$$= \frac{8\mu_o N I}{a\sqrt{125}} \qquad (2)$$

This equation should be used for calculating B in this experiment.

Procedure

Orientation of the e/m tube

1. The e/m tube and the Helmholtz coils are mounted on a low table for convenience. It is very important that the axis of the tube be oriented so that it is accurately parallel to the earth's magnetic field. Usually this has been done for you. A magnetic needle mounted on jewelled bearings is used as a compass or a dip needle and can be used to assist you in checking this alignment.

2. Rotate the graduated circle so that it is horizontal. In this position the instrument may be used as a sensitive compass. Place the magnetic needle on the stationary part of the base of the apparatus, near the front edge of the hinged board. Line up the two 90° marks on the graduated circle around the magnetic needle so that they are parallel with the edge of the hinged board, which, in turn, is parallel to the horizontal axis of the e/m tube. Orient the apparatus so that the e/m tube axis is accurately on the magnetic meridian.

3. Rotate the graduated circle so that it is vertical. You now have a sensitive dip needle. Place the magnetic needle on the top of the hinged board. Adjust the tilt of the board until the dip needle reads 90° on the graduated scale. This places the vertical axis of the axis of the e/m tube parallel to the earth's magnetic field. Clamp the coils securely in this position.
   The declination of the Earth's magnetic field at our location (43.47°N, 80.54°W) can be found at
   http://geomag.nrcan.gc.ca/apps/mdcal-eng.php

Adjustment of the anode Voltage ↓ **Adjustment of the magnetic field ↓**

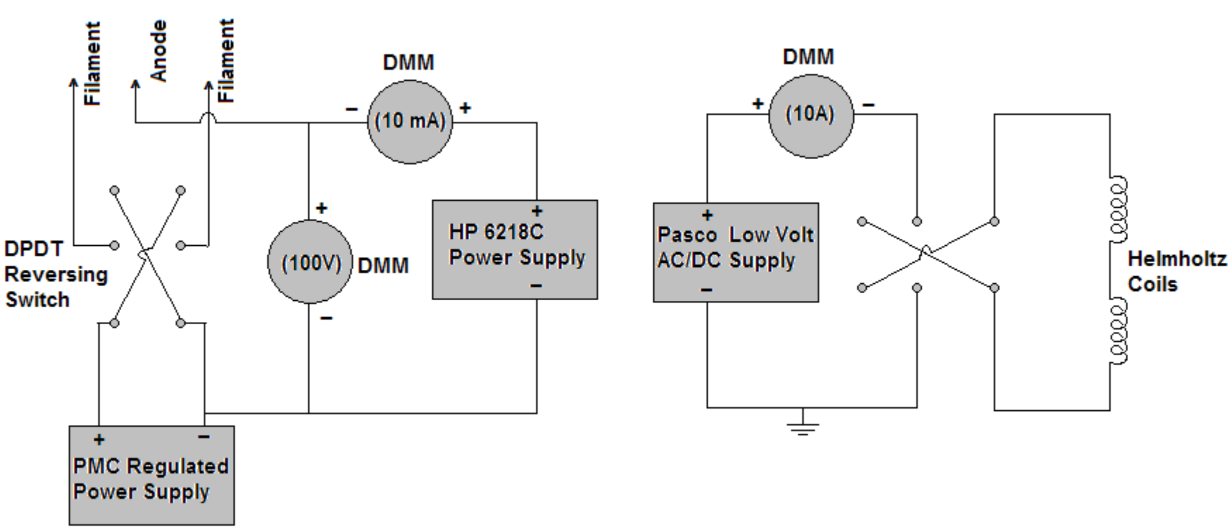


Figure 5

4. If not already done for you, wire up the circuits shown in Figure 5. The current from the Pasco low voltage power supplies the Helmholtz coils, and it can be adjusted by the controls on the front. The direction of the current should be such that the field from the coils opposes the earth's magnetic field. A reversing switch is provided for changing the direction of the current. Before turning anything on**, <u>have the circuit checked by an instructor</u>**.

5. Set the anode voltage, V, at a convenient value of 50 volts by adjusting the HP6218C supply. Turn on the filament (PMC regulated) supply and using **only the voltage control** on the front of the supply, set the current (as read on the front panel meter) to about 4A. Observe the anode current. If necessary, re-adjust the filament current (voltage control) until the milli-ammeter in the anode circuit reads 5 to 6 ma. **<u>Do not exceed 4.5 A filament current nor 6 mA anode current</u>!** The filament current supply has been currently limited to 4.5 amps. <u>Do not</u> adjust the current dial to circumvent this safety feature. Allow a 5-minute warm-up period so that a thermal equilibrium is reached.

<u>Compensation for Earth's Field</u>

6. Darken the room and observe the electron beam. Note that it is deflected south by the earth's magnetic field. From this observation what is the direction of the earth's magnetic field? The electron beam must be adjusted so that it comes out perfectly straight and horizontal (i.e. in the plane perpendicular to the axis of the Helmholtz coils). Reverse the filament current and note the effect on the beam. Explain what you observe.

   <u>With your fingertips</u> rotate the e/m tube so that the beam comes out horizontally. This should be done as accurately as possible and <u>with care</u> since there is a low pressure within the tube. It is best to have the filament current in such a direction that the slit in the anode cylinder sends light downwards away from the eyes of the observer.

7. Starting with 2.0 amps in the Helmholtz coils, observe the beam from above and see how the deflection (orbit size) changes as the current through the coils is changed. Observe also the effect of reversing the coil current. Account for your observations. Vary the anode potential and observe the effect on the beam. Account for your observations. Note that as the anode potential V is reduced the beam becomes sharper. Why? Reduce V until the beam is just seen to totally disappear just outside the anode slit. Note the V value at this point. What is the significance of this value of V?

   Set the anode potential to 20 volts and adjust the direction and magnitude of the coil current so that the beam (when viewed from above) is straight when referenced to a straight edge such as a ruler. At this point you have corrected for the earth's magnetic field. Record the coil current for 5 or 10 trials, and compute the magnitude of the average and the associated uncertainty of the earth's magnetic field. Section 10 contains the necessary data for this.

Taking Measurements

8. The current through the Helmholtz coils should be increased so that the electron beam describes a circle about 5 cm in diameter and passes between the cross bars. By the time the beam has travelled around the complete circle it is usually quite diffuse. If the proper adjustments have been made the centre of the beam will strike the centre of the back of the anode cylinder. Make a fine adjustment if necessary by rotating the e/m tube about its long axis.

9. Adjust the Helmholtz coil current so that the outside edge of the beam strikes the furthest one of the crossbars. The outside edge of the beam is used because it is described by the electrons which have the highest velocity. The electrons that leave the negative end of the filament fall through the greatest potential difference between the filament and the cylinder (anode), and thus have the greatest velocity. It is this potential difference which the voltmeter measures as it is connected in the circuit. Also, any electron which makes an ionizing collision with a mercury atom loses energy to the atom, and has its velocity reduced and so is bent into a circle of smaller radius by the magnetic field. Through further ionizing collisions, these electrons produce the general haze of light seen inside of the circle of the beam. Thus, the electrons which produce the outside edge of the beam at the selected bar on the staff (on the opposite side of the circle from the slit) are making their first ionizing collisions and have described a semicircle determined by the measured conditions of the experiment.

10. With V, the anode voltage, set at 20 volts, the current in the Helmholtz coils should be increased so the outside edge of the beam falls on the outside edge of each bar in turn. The parameter R in equation (1) can be calculated from the manufacturer's specification. They give the five values for D = 2R, which is identified in Figure 2, as 0.065, 0.078, 0.090, 0.103 and 0.115 meters respectively. To calculate B for each setting, you should use equation (2), using N = 72 and a = 0.33 meters.

    The values of I to be used in equation 2 are taken to be the current setting, I_D, less the current I_{earth} needed to annul the earth's field (from Section 7). Then $I = I_D - I_{earth}$ and $B = B_D - B_{earth}$.

    The product B^2D^2 for each setting should be calculated and the result should be independent of V (see equation (1)). Verify this and find an average value of B^2D^2.

11. Set V to 30 volts and check that the beam is horizontal. Rotate the tube if necessary. Then repeat Section 10 and find the average value of B^2D^2. Repeat for anode voltages at 40, 50 and 60 volts. Finally, plot a graph of 8V against the average values of B^2D^2 for each V. Do a linear least squares fit to the data to find the slope and use equation (1) calculate e/m. From this analysis, estimate the uncertainty in your e/m value, comparing this value to the accepted value e/m = 1.759×10^{11} coulomb/kg.

    You will see that this analysis increases the error at small values of I_D since it assumes that $(1 / B^2) = 1 / (B_D - B_{earth})^2$ in equation 1. This result becomes increasingly inaccurate when $I_D \rightarrow I_{earth}$ (or $B_D \rightarrow B_{earth}$) due to uncertainties in the measured values of both I_D and I_{earth}.
    Using realistic estimates of the uncertainties in I_{earth} and I_D, calculate the absolute error for each value of B_D and B_{earth} obtained from equation 2. Use these to find the absolute error in e/m for each value of B used in equation 1.
    Explain how this error influences the overall accuracy of your estimate of e/m.

<u>List of Equipment</u>

Welch 623 e/m tube with 623A Helmholtz coils
PMC Regulated Power Supply
HP 6218C Power supply (0-50V/0-.2A)
Pascoe Model SF-9584 low voltage AC/DC power supply
Data Precision Model 1350 Digital Voltmeter (Voltage Accuracy: ± (0.1% of reading + 1 l.s.d.)
Fluke Model 8050A DMM DCV Accuracy: ± (0.03% of reading + l.s.d.)
Nida Model 411 DMM (0-10A)
2 Reversing Switches
Low Table
Dip Needle