# A Guide to Waterloo Physics Lab Reports

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# Introduction

The following is meant as a general guide for practices to follow when writing a lab report for a University of Waterloo physics lab at the first, second, or third year level. This guide provides information on some basic concepts to keep in mind while writing your lab reports, an outline of the types of information to present in a report as well as how to organize it, a section describing how to deal with experimental errors, and finally a simple example lab report.

If any of the information in this guide conflicts with information given by your instructor in the lab course you should **always** follow the instructors advice.

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# 1 General Information

This section of the guide describes some general principles to keep in mind when writing a lab report as well as mentioning some good practices to follow in every section of the report.

## 1.1 Your Role

When writing a lab report you should pretend that you are a scientist conducting a peer review of a piece of scientific literature, typically the lab manual, by verifying or disproving the stated theory through measurement. This means that all calculations should be shown with full detail and that all results should be discussed and compared with accepted values from the literature with the end goal of deciding if the theory is sound or not.

Remember, science is advanced by disproving wrong theories more than by proving assumptions correct. Assume your audience is hostile to the ideas and analysis you are presenting. You must convince them you know what you are talking about.

## 1.2 Your Audience

Your report should stand **alone**. This means a student with a high school education should be able to follow your lab report without having access to the lab manual or any other supplementary sources. They may not understand all of the scientific terms or the equations they are reading but they should have a good idea of what you did and what you verified by the time they finish reading your report.

## 1.3 Typed or Not Typed?

Unless otherwise indicated by your instructor or lab manual the report can be hand written, typed, or a combination of both. It is common for students to type out the text and data tables of a report while leaving white space for the math so that it can be written in by hand after printing. Graphing can also be done by hand or computer. Both methods carry advantages and disadvantages.

The availability of these options depends highly on the course you are taking. There is typically no computer equipment available to type and print the report in first year labs whereas in upper year labs the reports are usually completed at home after a data collection only lab period. The main caveat to a hand written report is that it must be **clearly legible** to the marker. If the marker can not read your report it is considered wrong and you will lose marks.

## 1.4  Be Concise

Your lab report should be as concise as possible. This means making it as short as possible while presenting all of the required information to the audience in the simplest and clearest language you can. It is very common that students in lab courses will write large paragraphs in their reports that contain little to no information or that consist entirely of information already present previously in the report. This is a waste of both your time in writing the material and the markers time in reading it. It will cost you marks.

## 1.5  Quantitative vs Qualitative Observations

Observations made in lab experiments fall under the two general categories of qualitative and quantitative. A qualitative observation is an observation that relays some non-numerical property of your experiment to the audience. For example, "the desk is long" or "the sun is bright". A quantitative observation is an observation that relays a numerical property of your experiment. For example, "The table is 2.745m long" or "The wavelength of Sodium yellow light is 567.94 nm". Qualitative observations are often subjective and ill defined statements and should be avoided whenever a quantitative observation can be made.

It is common for students to default to qualitative comparisons when discussing results. For example, "Our result of 15.4 is close to the accepted value of 15.0." Since you have a value from your data and an accepted value you should do a quantitative comparison. For example, "Our result of 15.4 is 2.7 % higher than the accepted value of 15.0." The first statement is vague and can be interpreted differently by different audiences. The second statement clearly states how different the values are and acts as a good lead in to a discussion of the results. This is especially usefull when there are several results since having similar percent differences across multiple trials can indicate a systematic error in the data.

## 1.6  Citing Sources

When writing a report you should cite your sources. Especially when you use a source that is not the lab manual. Be specific when indicating where the information came from. If you are using an equation, state the equation as it appears in the source and then manipulate it to the form you want. Don't just say "using equation 6 from the lab manual" and write the final answer.

# 2 The Structure of a Lab Report

This part of the guide provides a description of the information you will present in a typical lab report. The information below is organized by section heading in this guide to make the guide more understandable. You should not blindly follow these section headings when planning your lab report. Do not be afraid to combine information from different sections if you **have a good reason to**. For example, it is often better to present all of the the raw data, calculations, and discussion under the single section heading of Analysis. Make a judgment call with the goal of presenting the information to the reader as clearly as possible.

For experiments that consist of several independent parts you should break the analysis into a separate section for each part. Each section should contain all of the measurements, analysis, and discussion related to that part. Each part should be given a descriptive label. Often using the section titles in the lab manual is enough but sometimes you will find them lacking the required detail. Simply writing "Part A" is not enough to identify a major part of your lab report. Use a title like "Part A: The Electrostatic Deflection of Electrons in a Cathode Ray Tube".

There is usually an over arching purpose for the lab that links the various parts and so a single purpose should be written at the beginning of the experiment. For the same reason the conclusions for the report should contain the results summary for all of the different parts in the same place at the end of the report.

## 2.1 The Header

At the top of the first page, or on a separate cover page, of your lab report you must have the following information:

1. The title of the experiment including the course code. For example, "Physics 111L Experiment 1: Measurement and Graphical Analysis" is a proper title.

2. Your FULL name.

3. Your student ID number.

4. Your partner(s) FULL name(s) (for all of your partner(s))

5. Your Presiding Instructor's name. Not the course coordinators name that appear's on the schedule of classes.

6. Your section number.

7. The date and time the experiment was performed. IE: 2:30 pm Thursday January 25, 2011.

## 2.2    The Purpose

The purpose is a **brief** paragraph describing the main purposes or goals of the lab. It should be short and generalized. A good purpose can often be as short as one or two sentences. Emphasis should be placed on the theoretical relations you will be testing. **Do not** summarize the theory of the experiment or include any of the procedure in your purpose. This section can always be completed before arriving to perform the experiment.

## 2.3    Pre-Lab Questions

Some experiments will have a set of questions or exercises that are expected to be completed before the lab time. **Clearly** label what you are doing when you complete these questions. Depending on the specific experiment it may be advantageous to answer the questions throughout the report in the sections they are related to or as part of a "Pre-Lab Exercises" section at the beginning of the lab. Make a judgment call. As with the purpose these questions can and should typically be completed before arriving to perform the experiment.

## 2.4    Recording and Presenting Raw Data

You must present your raw data in the form that you measured it so that the reader can examine your measurements themselves to verify your technique and judge the validity of your data. Avoid doing even simple calculations before reporting the data on your report. For example, when measuring a time value from an oscilloscope the time is calculated by measuring the number of horizontal divisions on the oscilloscope screen and multiplying by the Time Constant, TC. You should record both the TC and the horizontal distance measurement in the report and show the calculation of the time.

### 2.4.1    Taking a Measurement

For each measurement you make and record in your report you should check the following:

**Is the measurement properly and clearly labeled?**

You should have a clear label for your measurement. A measurement is useless to someone reading your report if the meaning of the number is not indicated somewhere. Don't re-use variable names for measurements of different quantities. Choose a different variable or add subscripts to differentiate them. Write a short sentence indicating which variable is which quantity even if the variable is given in the lab manual. For example, "The distance the cart traveled, d" defines the variable d for the rest of the report after this statement. Remember, the lab report should stand on it's own so you can't rely on the lab manual providing definitions.

**Are the units recorded properly?**

Without knowing the units of a measurement it is impossible to use it for any analysis. Be sure to indicate the units for both the measurement and the uncertainty. Generally measurements can be recorded in any units but must be converted to standard S.I. units for calculations. If the conversion to S.I. is common it is usually easier to do this as the numbers are recorded. For example, if you have a timer that reads in milliseconds you can convert the values to seconds in your head before recording the measurement. Sometimes it is best to use scientific notation to record measurements. If you have to use a calculator to do the conversion it is probably a calculation you should show in full detail on your lab report.

**Is the measurement recorded to the proper precision?**

The precision of the measurement is set by the limitations of the device used to take the measurement. For an analog device like a ruler or micrometer the measurement is usually read to $1/10^{\text{th}}$ the smallest increment indicated on the scale. For a digital scale simply copy the number indicated on the digital display.

When recording the measurement you must record significant zeros on the end. You must **NOT** drop the last digits even of they are zero because the number of digits you record indicate the precision of your measurement device. A measurement of 0.5 V is much different than a measurement of 0.500 V. The second indicates we could read the measurement device to the $1/1000^{\text{th}}$ order of magnitude while the first indicates a measurement device that is 100 times less precise was used in the lab.

**Is the measurement assigned the proper uncertainty?**

A detailed guideline for determining the uncertainty in a measurement is given in section 3.2.3. A summary of the most common steps for determining your uncertainty values is provided below.

1. Check if the uncertainty for your measurement device is listed somewhere such as the lab manual or on the device. If not move to step 2. Otherwise jump to step 3.

2. Assign an uncertainty of half the smallest scale on your device.

3. Decide if the uncertainty should be raised to account for experimental errors in the procedure or apparatus. You **must** provide a justifying statement if you choose to raise the uncertainty from either steps 1 or 2.

4. If you cannot assign an uncertainty with the previous steps then consider a statistical approach. You will need to make this decision in the lab so that you can make the required measurements before leaving. The statistical method is outlined in section 3.2.4.

### 2.4.2 Recording Measurements in Tables

When taking several measurements that are the same it is almost always advisable to summarize them in a table. Give the table a clear label and caption so that you can reference it in your discussion. You can save time and space by writing the name, variable, units, and uncertainty for each group of the same quantity once at the beginning of a row or column instead of repeating them next to each value in the table. If the uncertainty is different for each row or column give it its own separate and clearly labeled row or column. Remember that since the report should be understandable for someone without the lab manual the data should be clearly labeled. Don't just use the variable names in the lab manual. For example, when recording distance measurements in a table, "d, m" is not as good as "Distance Traveled, d (m) ± 0.005m".

Typically when someone wants to quickly extract data from a report they will go straight to the tables with minimal to no reading of the text of the report. To facilitate clarity you should put a detailed column heading even if the variables are defined elsewhere in the report.

## 2.5  Performing and Reporting Calculations

As part of justifying to the audience that you are a competent scientist you must detail how you convert your raw data, both quantitative and qualitative observations, into meaningful results. This will typically involve converting large sets of measurements into a few or even a single final result such as a measurement of the permittivity of free space. This section of the guide details the calculation steps you should show as justification of your final results.

### 2.5.1  Steps to Show in a Calculation

Before starting the calculation you wish to perform you should decide whether the value you are after is a final result or an intermediate step. The final result will typically be calculating something like the gravitational acceleration of masses near the Earth's surface, g, so that it can be compared with the accepted value. An intermediate step is any calculation that does not end with a final result value.

When performing any calculation you should show all of the following steps:

1. State the full equation(s) and quote the source(s) (IE: equation 6 from experiment 2 of the physics 260A lab manual). The equations should not be buried in the middle of your text. Put each equation on its own line.

2. Show any rearranging of the equation(s) to get them into the form you need for your analysis.

3. Show a calculation step by writing the values of each variable with its units but NOT the uncertainty.

4. Perform the calculation and record the result. Keep lots of extra significant figures in your result even if one of the numbers used to get the result contains fewer significant figures. For example, $(3.1)(2.9783) = 9.23273$ not $9.2$.

5. Next to the calculation you should derive the equation for the uncertainty of your answer.

6. Show a step plugging in the values to the uncertainty equation you have derived.

7. Record the resulting uncertainty with lots of significant figures. Remember the units.

When performing a final result calculation you should follow all of the above steps and additionally show the following:

1. Round the uncertainty to one significant digit unless the first digit is a 1 then keep two significant digits.

2. Round your final answer to the same precision as the uncertainty. The measurement should **NEVER** be quoted to a higher precision than the uncertainty. If you do this you are saying that you can confidently report the precision to, for example, mm while simultaneously saying you are unsure of the value on the order of cm.

3. Write the final result in the form $K = K_{\text{experimental}} \pm \Delta K$ where $K_{\text{experimental}}$ is the final result you calculate from your measurements and $\Delta K$ is the **absolute** uncertainty in your final result. Final results should always be quoted with absolute uncertainty. Remember to include the units on the experimental value and its uncertainty.

### 2.5.2 Recording Calculations in Tables

When performing multiple calculations of the same quantity it is almost always advisable to summarize them in a table. Use the same guidelines for formatting and labeling the table as outlined earlier in section 2.4.2. In addition to the guidelines for measurement tables you are required to show a sample calculation for **every** unique calculation in the table. This is typically accomplished by performing all the calculations required to fill in the cells of the table as you move across a row or down a column. These calculations are written directly under the table with a heading such as "Sample calculations for table 1 using row 2". Remember to show the uncertainty calculations as well.

It is typically more convenient to report the measurements and calculations together in a single table rather than having several tables within each section of a report. This condenses all of the relevant information into a convenient and easy to read form for your audience. If the table spans multiple pages or the number must be printed so tiny as to be illegible then you should consider using multiple tables. Use your best judgment when designing your report.

## 2.6 Plotting a Graph

Plotting your data in an easy to read graph is one of the most time consuming but also one of the most important skills you will learn from your physics lab courses. Graphs provide an excellent tool to represent large quantities of complicated data in a far more intuitive form than a list of numbers in even the most well made table. When plotting a graph you should consider the following:

**Sizing the graph on the page**

Make your graphs as large as possible on a single page of your report. The larger the graph the easier it is to see the details of the data and to make measurements. This is especially important for hand drawn graphs and graphs that you will have to read data from after you have plotted them.

**Choosing a title**

The title of your plot should convey what you are plotting and why you are plotting it. A title like "x vs. t" is useless from the point of view of your audience. A proper title would be "Figure 1: The Distance Traveled by an Air Cart, x (m), versus the Time Taken to Traverse the Distance, t (s), to Determine the Gravitational Acceleration of Masses Near the Earth's Surface". This title conveys exactly what the plot is while also providing an easy reference label (Figure 1) that you can use in your discussion. Under the title you should write the variables you are plotting in the form "x vs. t".

**Choosing an axis scale**

The goal when choosing an axis scale is to use as much of the graph paper as possible to show your data without using an unconventional scale. To set a conventional axis scale simply look at your data and find the range that the dataset covers. Choose nice values for the max and min of your axis that encompass the range of values you wish to plot. Divide the difference of the max and min values by the number of major divisions on the graph paper to calculate a spacing for the plot. You should adjust your scale to a reasonable spacing if the above calculation results in an inconvenient number. For example, if you calculate that your spacing should be 1 division = 3.45643 meters you should round it up to a nicer number. Remember to check that the full dataset still fits into the scale you will end up with after the rounding. Avoid ratios that result in conversion factors that never end like 1:3 = 0.33333333 per division on the plot.

Plots **DO NOT** have to start at zero on the origin. There is nothing special about the lower left corner of the paper.

**Labeling the axis**

You must label each axis clearly with a description of the variable plotted and its units. Don't just use a label like "x". Use "Distance Traveled, x (m)".

**Adding a legend**

If you are plotting more than one data series on the same plot you should include a legend that indicates which line of the plot is which set of data. This is typically done using different colors and/or symbols for each dataset. Avoid colors if your report will be printed in black and white.

**Plotting the data**

To plot the data simply put you chosen symbol at each of the points required by your dataset. The symbol should be small enough to indicate only the point you are trying to plot but large enough to be seen even if you have to write over the point as part of the analysis.

**Calculating the slope**

Typically the slope calculation can be done on the plot itself. Follow the rules for calculations outlined earlier in this guide. In addition you should clearly indicate on the best-fit line which points you are using for the slope calculation. Remember to clearly indicate which calculation applies to which line if there is more than one. The points used to calculate the slope should be chosen as far from each other as possible on the best-fit line. You should **never** choose a point that is included in your raw data even if the best-fit line you draw appears to overlap it perfectly.

For graph paper with linearly scaled axes the slope is calculated with the equation,

$$S = \frac{y_2 - y_1}{x_2 - x_1} \tag{2.1}$$

For graph paper with logarithmic axes use the equation,

$$S = \frac{\log(y_2) - \log(y_1)}{\log(x_2) - \log(x_1)} \tag{2.2}$$

**Dealing with outliers**

If there is a point that is clearly outside the trend set by the rest of your data then you can typically ignore it when analyzing the plot. You must still plot the point and clearly indicate in your report that you consider this point a statistical outlier and provide some justification for that in your discussion of the plot.

**Uncertainty Analysis of a Plot**

The procedure for representing uncertainty on a plot is detailed in the uncertainty section of this guide, section 3.4. This section also discusses calculating uncertainty in the slope of your graphs and how to judge the uncertainty in a point that you read from the graph.

## 2.7　The Discussion

Ultimately the goal of a scientific paper is to convey your analysis of a theory or experiment to other scientists in a way that they will agree with your ideas whether those ideas support or contradict the established theory of your research topic. To this end you must provide discussion throughout the report to support your analysis. The discussion serves several purposes including but not limited to justification of measurement techniques, justification of uncertainty values, description of the theory, analysis of final results, and suggestions for improving the experiment.

The discussion is typically the most difficult and worst written part of a lab report. To write a proper discussion you must understand the theory of the experiment you are performing, what measurements you took and why, and what the analysis you are performing means. Discussion points should be placed throughout the report as needed to justify your analysis. Try to place them near the topic being discussed. It is frustrating to your audience to have to flip through several pages of a report to look at a data table or plot and the discussion that goes with them because the author chose to place all discussion at the end of a report.

Your discussion points should each follow the outline of Statement -¿ Analysis -¿ Conclusion. The statement is a short sentence or two that tells the audience what your observations are and how well they compare to the expected behavior. Avoid vague statements such as "my value of g is close to the theoretical value". Different people can have a different understanding of "close". Be as specific and quantitative as possible. A much better statement would be "Our measured value of acceleration of objects near the Earth's surface, $g = 10.4m/s^2$, is 6% higher than the accepted value of $9.8m/s^2$". A common mistake made in undergraduate labs is to write a discussion that consists entirely of these observations. Any five your old could come into the lab and repeat what he (she) sees happening in front of them. As a scientist your job is to tell the audience what these observations mean.

To this end the statement is then followed by a discussion to tell your audience exactly what you think your results mean. Avoid vague statements and make sure your stated reasons match the actual results you have measured. A statement such as "friction caused my measured value of gravitational acceleration to be $25m/s^2$" is bad in two ways. Firstly, you should indicate what objects the friction is between. Secondly, the stated reason (friction that reduces acceleration) does not match the outcome (a higher acceleration). When writing the discussion you should consider the following questions:

1. How well does the experimental station match the assumptions that underpin the theory for the experiment? If there are differences how would they effect the final results?

2. Was there anything about the equipment or measurement techniques that would have introduced error into your results? Was the error systematic or random? Is the random error accounted for in the uncertainty? Why not?

   Note: If you recorded a source of error to increase your uncertainty values and the final result was outside the uncertainty range from the accepted value you **cannot**

restate the same errors over again in your discussion. You are effectively saying the uncertainty you chose was too low. In that case you should go back, increase it and redo all of the related uncertainty calculations in the lab report.

3. Can these effects explain the percent difference you calculated? How? Be specific and reference the theory and your measurement techniques when trying to answer this.

4. If there are no identifiable systematic errors and the random errors do not account for differences from the theory then is the theory wrong? This is highly unlikely in your undergraduate experiments and you should only use this discussion as a last resort. Remember, the instructor can probably think of errors you missed.

Finally you should state the conclusion you reached from your observations and your analysis by provide a concluding statement for each of your your discussion points. This statement is a brief summary of the previous discussion such as "The time values measured with our stop watch are systematically too large and this has caused our final result to be outside the uncertainty range calculated".

## 2.8   The Conclusion

At the end of the lab report there should be a conclusion section. The conclusion is separate from your previous discussion and summarizes the main results for the entire report including your quantitative results **and** their uncertainties. Do not repeat your discussion or perform new analysis here. This section should answer the question "How well did I achieve or not achieve the goals set out in my purpose". This section should be brief because all of the analysis is already done in the discussion section.

# 3    Error Analysis

This section of the guide discusses the phenomena of errors in your experiments and how you should deal with them as part of a proper scientific analysis. When performing an experiment there are two types of errors that can affect your results, systematic and random.

## 3.1    Systematic Errors or Bias

A systematic error is an error that biases all of your results in the same way. These errors tend to result from bad assumptions in the theory compared to the equipment available, broken or mis-calibrated equipment, or a bad technique for gathering the raw data. There are several ways to account for systematic errors in your experiments such as but not limited to redesigning the data gathering techniques to eliminate the error, performing the experiment multiple times with different sets of equipment, or designing a new experiment meant to determine the size of the bias so that you can adjust your results accordingly. Whichever method you choose to employ you should show every step and explain exactly what you are doing and why in your lab report. Remember, scientists design experiments and document their techniques so that they are understandable and reproducible to other scientists. Some examples of systematic errors you may encounter are,

1. Neglect of friction on an air track. Often there is a small amount of friction between the cart and the air cushion it rides on for any air track that is ignored in the development of the theory of an experiment. This is a common assumption in all first year air track labs. Even if the friction is small it will still bias all of the accelerations and velocities to be lower than theory says they should be.

2. Timing something with a manually operated stop watch. There will almost always be a slight delay between the event being timed and the reaction of the person operating the stopwatch timing the event. If the operator of the stopwatch is always too eager or too cautious when using the device your times will always be biased too high or low.

3. Slippage in mechanical dials. Often when using a measurement device in the lab the measurement scale will be printed on a mechanical dial that has slipped over time. The common micrometer used in your experiments can have this problem. When the jaws of the device are perfectly closed the reading is not exactly zero. If you just take the dial reading directly all of your results will be too high or low depending on where the true zero is.

4. Printing errors on analog scales such as common rulers or the gridlines on graphing paper can cause the scale to indicate the wrong value. If the ruler scale was printed stretched then all measurements would be too low.

## 3.2 Random Errors or Uncertainty

### 3.2.1 The General Idea

Every measurement you make in a laboratory experiment will have some amount of random error resulting from one or more sources. The predominant source of this type of error you will encounter in your undergraduate physics experiments are limitations of the measurement devices in the labs. Consider for example a simple meter stick with a smallest scale increment of one millimeter. In an ideal situation it is easy to make measurements to the millimeter and it is possible to estimate measurements to the $1/10^{th}$ of a millimeter by estimating the distance between adjacent smallest increments. It is impossible to be exactly certain of your estimate and so you should assign an uncertainty range to the measurement.

The measured quantity is then expressed as $x \pm \Delta x$, where the range $x + \Delta x$ to $x - \Delta x$ represents the range within which the observer believes, with reasonable confidence, the "true" value lies. One might assign a probability distribution to the true value of the quantity. The center of the distribution would be x, and $\Delta x$ would be a measure of the width of the distribution. The simplest kind of distribution would be a rectangular one of width $2\Delta x$, as in figure 1a. A more realistic distribution would be similar to figure 1b, the familiar "normal" distribution curve.



| | |
|---|---|
| (a) Rectangular Probability Distribution | (b) Normal Probability Distribution |
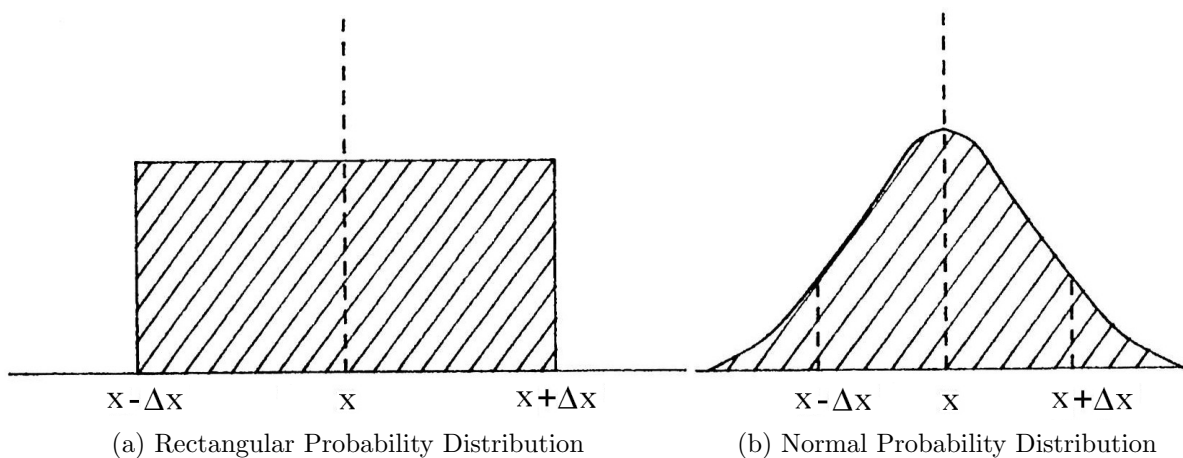
Figure 1: Example Probability Distributions

Figure 1a implies that the true value is equally likely to be any value within the error limits, and cannot be outside them. Figure 1b implies that the true value is most likely to be at the measured value, but has some probability of being outside the error limits, but not far outside.

When we add two measurements, how do we calculate the error of the sum? Let us consider $x_3 \pm \Delta x_3 = (x_1 \pm \Delta x_1) + (x_2 \pm \Delta x_2)$. The most simple-minded thing to do is to simply add the errors $\Delta x_3 = \Delta x_1 + \Delta x_2$. This, however, is unrealistic. It gives us an extreme

limit of error and in any conceivable physical situation overestimates the error. The error in the sum may be determined by taking the convolution of all possible true values of $x_1$ with all possible true values of $x_2$. The convolution is a summing of the product of all the probabilities for values of $x_1$ and $x_2$ which combine to give $x_3$. This convolution is the true value probability distribution for $x_3$. The convolution for two rectangular distributions is illustrated in figure 2.
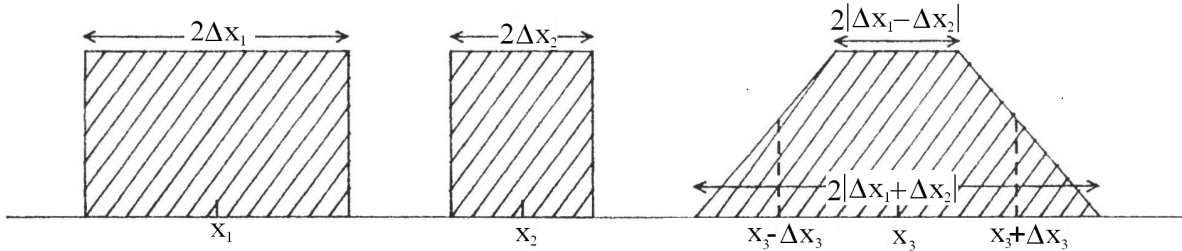


Figure 2: Convolution of Rectangular Probability Distributions

Although the full width of the distribution is given by the sum of the errors, the width at half maximum, which is a more realistic width, is a smaller quantity.

If one assumes the normal distribution (figure 1b), and combines two measurements, one again finds that the width at half maximum is less than the sum of the individual widths, as shown in figure 3.
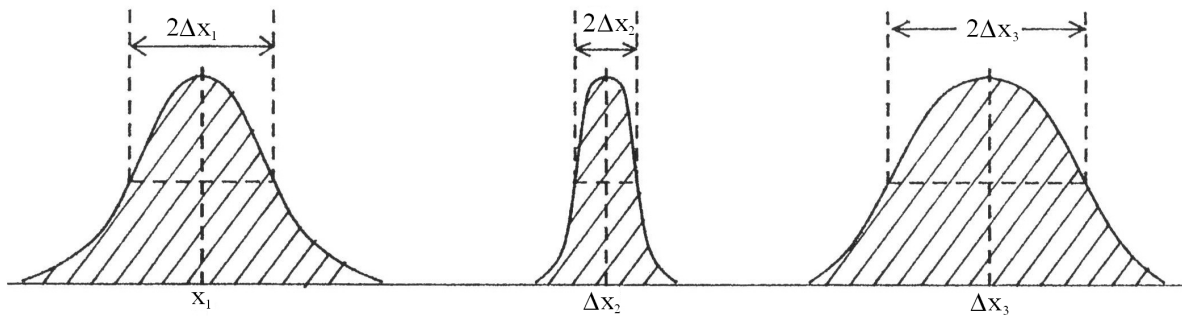


Figure 3: Convolution of Normal Probability Distributions

In fact, the value of $\Delta x_3$ (half width at half maximum) is given by $\Delta x_3 = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2}$. This result is exact for the normal or Gaussian curve, and is the best approximation to use when the exact probability distributions are not known. It should be noted here that this method assumes that the uncertainty values are completely independent and non-correlated. If there is correlation between the probability distributions then correction terms must be added to the uncertainty expression. The specific mechanics of this are beyond the scope of your lab courses at Waterloo but be aware of these considerations.

### 3.2.2  Uncertainty Types and Representations

Uncertainty is always represented in one of three ways,

1. Absolute Uncertainty: Absolute uncertainty is the absolute range of your uncertainty in the units of the measurement. For example, $x = 14.56m \pm .08m$ where $0.08m$ is the absolute uncertainty. The uncertainty in your results should always be reported in absolute uncertainty unless you are told otherwise in the lab manual or by your instructor.

2. Relative Uncertainty: Relative uncertainty is the ratio of absolute uncertainty to the value of the measurement. For example, if $x = 14.56m \pm .08m$ then the relative uncertainty is simply $\frac{0.08m}{14.56m} = 0.005495$. Relative uncertainty is **always unitless**.

3. Percent Uncertainty: Percent uncertainty is 100 times the relative uncertainty. So, continuing our example from above, the percent uncertainty will be 0.5495%. Remember to always include the % unit on the numbers to clearly indicate what you are reporting to the reader and avoid confusion with relative uncertainty values. Note: Percent uncertainty is sometimes refered to as percent error. Since the term percent error can refer to either the percent uncertainty or the percent difference you should **never** use it to avoid confusing the reader.

So far in this guide all uncertainties are represented using the notation $\Delta(variable)$ (ex: $\Delta x$ or $\Delta d$). Sometimes this notation can be inconvenient to use, especially if you have a lot of $\Delta$'s in your equations. Some common alternate notations are shown in table 1. Choose **ONE** notation and stick with it in each report. **Do not mix styles**. **Never** represent your uncertainty values without clearly labeling them with the variable they are associated with.

| Style | Absolute | Relative | Percent |
|-------|----------|----------|---------|
| 1 | $\Delta x$ | $\frac{\Delta x}{x}$ | $\% \Delta x$ |
| 2 | $\epsilon_x$ | $\frac{\epsilon_x}{x}$ | $\% \epsilon_x$ |
| 3 | $e_x$ | $\frac{e_x}{x}$ | $\% e_x$ |
| 4 | $\sigma_x$ | $\frac{\sigma_x}{x}$ | $\% \sigma_x$ |

Table 1: Uncertainty Notation Schemes. Avoid use of scheme four as it can very easily be confused with standard deviation.

### 3.2.3  Assessment of Uncertainty

Every measurement you take will have at least some uncertainty that **must** be indicated in your report. The uncertainty is dictated by the device used to take the measurement and the situational limitations the measurement is performed under. For both digital and analog scales the first method to find the uncertainty is to simply check if the value is written somewhere for you to copy. Typically it would be written on the device itself or in the lab

manual. Don't forget to check the equipment list for your experiment. If the value is not available you must use your best judgment to decide on a value. A general rule of thumb for an analog or a digital scale is to start at half the smallest increment on the scale.

Example 1: A meter stick is graduated in mm and so the uncertainty will be $\pm 0.5mm$. Example 2: A digital multimeter with a smallest increment of $5V$ would have an uncertainty of $\pm 2.5V$.

Occasionally the equipment or procedure you are following in the lab will increase the uncertainty in your measurements. Some sources of error that could cause this include a faded or hard to read scale on the measurement device or a large amount of noise in an electrical signal that makes the measured values fluctuate on your meter. In these cases you should assign a reasonable uncertainty and explain your choice in a short sentence or two. This explanation should be provided where you record the uncertainty the first time in the report. In cases where you absolutely cannot assign an uncertainty using the above methods you should consider using the statistical approach outlined below in section 3.2.4.

Experimental uncertainty assessment is important. Be realistic and honest. Don't expect experimental uncertainties to provide you with a way of excusing sloppy work. It is as bad to overestimate experimental uncertainties as to underestimate them. Example uncertainty determinations are provided in the example lab in section 4.

### 3.2.4 Assessment of Uncertainty with Standard Deviation

If you cannot determine an uncertainty value from the methods in section 3.2.3 then a value for the uncertainty can be estimated by taking several repeated measurements and finding the average and standard deviation. To ensure a reasonable estimate and reduce the effects of outlier data you should take ten or more measurements. The average, $\overline{X}$, of a set of n measurements, $x_i$, is calculated in the standard way according to equation 5.3. The standard deviation, $\sigma_X$, of a value as calculated from a sampling of a population is found according to the equation below.

$$\sigma_X = \sqrt{\frac{1}{n-1} \sum_i^n (x_i - \overline{X})^2}$$ (3.1)

The uncertainty in the average value is then given by the equation below.

$$\Delta x = \frac{\sigma_x}{\sqrt{n}}$$ (3.2)

## 3.3 Propagation of Uncertainty through Calculation

### 3.3.1 General Rules

When you compare your results with handbook values remember these points: **The handbook values are not sacred**. The difference between your result and the handbook result is not an error - it is a discrepancy. Your experimental uncertainty in your result may account for the discrepancy, or it may not. There may be systematic errors in the experiment (badly calibrated voltmeter, for example) that you cannot assess. You may or may not be able to come up with plausible explanations for the discrepancies.

Uncertainties should never be quoted to more than two figures, usually just one. Quantities should never have digits extending beyond the error limit. Thus:

$15.47206 \pm 3.325\%$ should be expressed as $15.5 \pm 3\%$ or $15.5 \pm .5$

Note that you should use the extra figures in a % uncertainty to calculate the absolute uncertainty, and vice-versa, or to carry along into later calculations, but they should not appear in your final quoted result.

### 3.3.2 Addition and Subtraction

When propagating uncertainty through addition or subtraction calculations the **absolute** uncertainty in the result is the square root of the sum of the squares of the individual **absolute** uncertainties for each measurement, $x_i$. Remember the measurements must be independent! As an equation this is,

$$\Delta X = \sqrt{\sum_{i=1}^{N} (\Delta x_i)^2} = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 + \ldots} \qquad (3.3)$$

**Example:** Calculate d and its uncertainty for $d = a + b - c$ when

$a = 15.4m \pm 0.20m$ $\qquad\qquad$ $b = 2.22m \pm 0.05m$ $\qquad\qquad$ $c = 10.58m \pm .15m$

Answer:

$d = a + b - c$ $\qquad\qquad\qquad\qquad$ $\Delta d = \sqrt{(\Delta a)^2 + (\Delta b)^2 + (\Delta c)^2}$

$\quad = 15.4m + 2.22m - 10.58m$ $\qquad\qquad$ $= \sqrt{(0.20m)^2 + (0.05m)^2 + (0.15m)^2}$

$\quad = 7.04m$ $\qquad\qquad\qquad\qquad\qquad$ $= 0.25495m$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $= 0.25m$

Thus $d = 7.04m \pm 0.25m$.

### 3.3.3 Multiplication and Division

When propagating uncertainty through multiplication or division calculations the **relative** or **percent** uncertainty in the result is the square root of the sum of the squares of the individual **relative** or **percent** uncertainties for each measurement, $\frac{\Delta x_i}{x_i}$. The measurements must be independent! As an equation this is,

$$\frac{\Delta X}{X} = \sqrt{\sum_{i=1}^{N} \left(\frac{\Delta x_i}{x_i}\right)^2} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \left(\frac{\Delta x_2}{x_2}\right)^2 + \left(\frac{\Delta x_3}{x_3}\right)^2 + \dots} \qquad (3.4)$$

**Example:** Calculate the result and relative uncertainty for $d = \frac{ab}{c}$ where,

$a = 15.4m \pm 0.20m$ $\qquad\qquad$ $b = 2.22m \pm 0.05m$ $\qquad\qquad$ $c = 10.58m \pm .15m$

Answer:

$$d = \frac{ab}{c} \qquad\qquad \frac{\Delta d}{d} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2 + \left(\frac{\Delta c}{c}\right)^2}$$

$$= \frac{(15.4m)(2.22m)}{10.58m} \qquad\qquad = \sqrt{\left(\frac{0.20m}{15.4m}\right)^2 + \left(\frac{0.05m}{2.22m}\right)^2 + \left(\frac{0.15m}{10.58m}\right)^2}$$

$$= 3.2314m \qquad\qquad\qquad = 0.0296$$

$$\qquad\qquad\qquad\qquad = \pm 3.0\%$$

Thus $d = 3.2314m \pm 3.0\%$ or in absolute uncertainty $d = 3.2m \pm 0.1m$.

### 3.3.4 Error Measurements in Functions

There are two methods to estimate the error in a function.

**Method 1: Limiting Values**

This method evaluates the error, $\Delta f(x)$, in a function $f(x)$ by taking half the difference between the maximum and minimum value of the function. As an equation this is,

$$\Delta f(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2} \qquad (3.5)$$

## Method 2: The Analytic Method

For a function such as $y = f(x)$ the analytic method uses the fundamental result of calculus:

$$\delta y = \frac{df(x)}{dx}\delta x$$

where $\delta x$ and $\delta y$ represent the (small) variations in x and y which can substituted with the uncertainties, $\Delta x$ and $\Delta y$ in those quantities:

$$\Delta y = \frac{df(x)}{dx}\Delta x \tag{3.6}$$

**Example 1:** What is the uncertainty for $f(x) = \sin(x)$ if we have $x = 49° \pm 7°$?

Answer:

| **Method 1** | **Method 2** |
|---|---|
| $\Delta(\sin(x)) = \dfrac{\sin(x+\Delta x) - \sin(x-\Delta x)}{2}$ | $\Delta(\sin(x)) = \dfrac{d\sin(x)}{dx}\Delta x$ |
| $\quad = \dfrac{\sin(49° + 7°) - \sin(49° - 7°)}{2}$ | $\quad = \cos(x)\Delta x$ |
| $\quad = \dfrac{\sin(56°) - \sin(42°)}{2}$ | $\quad = \cos\left(49°\dfrac{\pi}{180°}\right)\left(7°\dfrac{\pi}{180°}\right)$ |
| $\quad = 0.079953483$ | $\quad = 0.080152731$ |
| $\quad = 0.08$ | $\quad = 0.08$ |

**Note:** When dealing with the uncertainty of trigonometric functions in method 2 the angles that you sub into the equations **must** be in Radians.

**Example 2:** What is the uncertainty for $f(x) = \ln(x)$ if we have $x = 22cm \pm 3cm$?

Answer:

| **Method 1** | **Method 2** |
|---|---|
| $\Delta(\ln x) = \dfrac{\ln(x+\Delta x) - \ln(x-\Delta x)}{2}$ | $\Delta(\ln x) = \dfrac{d\ln(x)}{dx}\Delta x$ |
| $\quad = \dfrac{\ln(22cm + 3cm) - \ln(22cm - 3cm)}{2}$ | $\quad = \dfrac{1}{x}\Delta x$ |
| $\quad = \dfrac{\ln(25cm) - \ln(19cm)}{2}$ | $\quad = \dfrac{3cm}{22cm}$ |
| $\quad = 0.137218423$ | $\quad = 0.136363636$ |
| $\quad = 0.14$ | $\quad = 0.14$ |

### 3.3.5 Powers and Roots

Suppose x is raised to the power n : $y = x^n$. Using our result from method 2 above, we have: $\Delta y = nx^{n-1}\Delta x$ which can be re-arranged to give:

$$\frac{\Delta y}{y} = |n|\frac{\Delta x}{x} \quad \text{or} \quad \%\Delta y = |n|\,(\%\Delta x) \tag{3.7}$$

Thus, the percentage (or relative) uncertainty in $y$ is just $|n|$ times the percentage (or relative) uncertainty in $x$.

**Examples:** Find $y = x^n$ and its uncertainty for,

a) $x = 15.40m \pm 1.6\%$ and $n = 2$

$$y = x^n \qquad\qquad\qquad\qquad\qquad \%\Delta y = |n|\%\Delta x$$
$$y = (15.40m)^2 \qquad\qquad\qquad\qquad = (2)(1.6\%)$$
$$y = 237.16m^2 \qquad\qquad\qquad\qquad = 3.2\% = 3\%$$

Thus $y = 237.16m^2 \pm 3\%$ or in absolute uncertainty $y = 237m^2 \pm 7m^2$.

b) $x = 53.270s \pm 0.007s$ and $n = \dfrac{7}{12}$

$$y = x^n \qquad\qquad\qquad\qquad\qquad \frac{\Delta y}{y} = |n|\frac{\Delta x}{x}$$

$$y = (53.270s)^{\frac{7}{12}} \qquad\qquad\qquad\qquad = \left|\frac{7}{12}\right|\frac{0.007s}{53.270s}$$

$$y = 10.16517609s^{\frac{7}{12}} \qquad\qquad\qquad = 0.000401698 = 0.04\%$$

Thus $y = 10.165s^{\frac{7}{12}} \pm 0.04\%$ or in absolute uncertainty $y = 10.165s^{\frac{7}{12}} \pm 0.004s^{\frac{7}{12}}$.

### 3.3.6 Multi Step Calculations

When doing a multi-step calculation you can calculate the uncertainties for each step separately and carry them along. For example,

$F = \sin(ab)\sqrt{\frac{c+d}{e}}$

will have uncertainty,

$$\frac{\Delta F}{F} = \sqrt{\left(\frac{\Delta\sin(ab)}{\sin(ab)}\right)^2 + \left(\frac{1}{2}\frac{\Delta\left(\frac{c+d}{e}\right)}{\left(\frac{c+d}{e}\right)}\right)^2}$$

$$= \sqrt{\left(\frac{\cos(ab)\Delta(ab)}{\sin(ab)}\right)^2 + \left(\frac{e}{2(c+d)}\Delta\left(\frac{c+d}{e}\right)\right)^2}$$

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$$= \sqrt{\left(\frac{\cos(ab)\Delta(ab)}{\sin(ab)}\right)^2 + \left(\frac{e}{2(c+d)}\frac{c+d}{e}\sqrt{\left(\frac{\Delta(c+d)}{c+d}\right)^2 + \left(\frac{\Delta e}{e}\right)^2}\right)^2}$$

$$= \sqrt{\left(\cot(ab)(ab)\sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}\right)^2 + \left(\frac{1}{2}\sqrt{\left(\frac{\sqrt{(\Delta c)^2+(\Delta d)^2}}{c+d}\right)^2 + \left(\frac{\Delta e}{e}\right)^2}\right)^2}$$

$$= \sqrt{\left((ab)\cot(ab)\sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}\right)^2 + \frac{1}{4}\left(\frac{(\Delta c)^2+(\Delta d)^2}{(c+d)^2} + \left(\frac{\Delta e}{e}\right)^2\right)}$$

### 3.3.7 Errors in Measurements of a Special Nature

Example: You are trying to measure the spacing "d" of a periodic pattern such as a grating or diffraction pattern, and you have many measurements ( $x_n$ values) of the "spot" positions.

| Measurement Number, $i$ | Measurement, $x_i \pm 0.05mm$ |
|---|---|
| 0 | 1.25 |
| 1 | 1.94 |
| 2 | 2.66 |
| 3 | 3.39 |
| ... | ... |
| 12 | 9.63 |

Table 2: Example Special Measurements Data

Taking the difference we get $x_{12} - x_0 = 9.63mm - 1.25mm = 8.38mm$ with uncertainty $\sqrt{0.05mm^2 + 0.05mm^2} = 0.07mm$.

Dividing by 12 (spacings), we have $d = 0.698 \pm 0.006mm$. Thus, while the individual spacings were measured to $\pm 0.07mm$, the average spacing is accurate to $\pm 0.006mm$. However, it is also important to closely examine each reading $(x_n)$ and check the individual differences $(x_n - x_{n-1})$ to be sure that you have not missed any "spots".

Note also that a number of sets of $(x_{12} - x_0)$ would be required to statistically reduce your error limits. The error can be reduced by a factor of $\frac{1}{\sqrt{n}}$ where n is the number of independent measurements of the quantity.

## 3.4   Uncertainty in Graphs

This section describes various methods to determine the uncertainty in data read from the graphs you will make in almost every lab you perform. Plotting and labeling your graphs is discussed in section 2.6.

### 3.4.1   Error Bars

Error bars are lines drawn horizontally and/or vertically out from each of the data points on your plots. The error bars directly represent the uncertainty in each of your data points. As such, the length of the error bars is determined by the **absolute** uncertainty of the x and y coordinates of each data point. The variable plotted on the x-axis determines the length of the horizontal bars and the y-axis determines the length of the vertical bars. The bars do not have to be symmetric about the points. See figure 4 for a depiction of error bars. Note: If the error bars are so small that they won't be visible on the graph they can be excluded.



(a) Straight Line Data                    (b) Parabolic Data
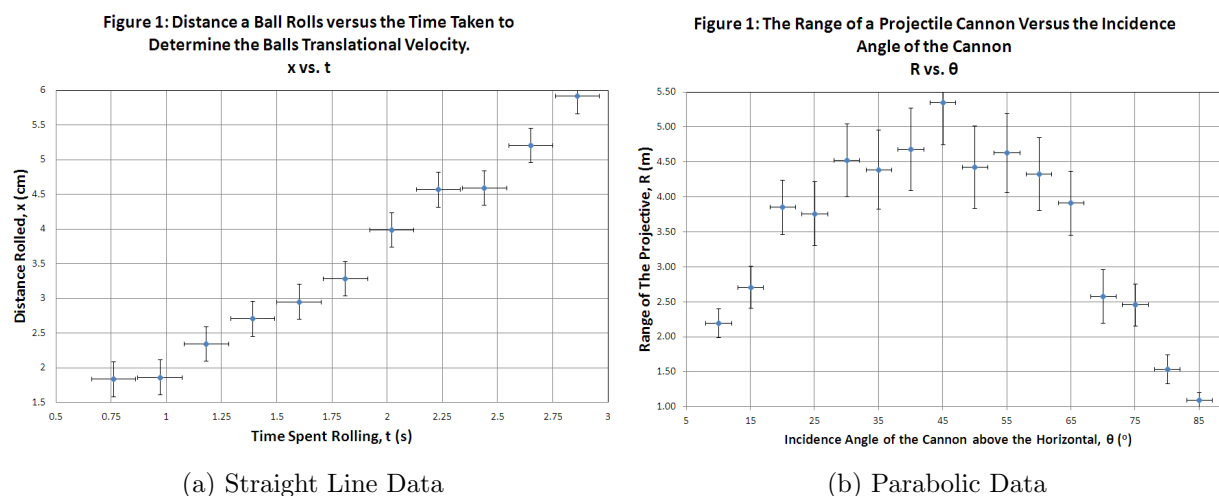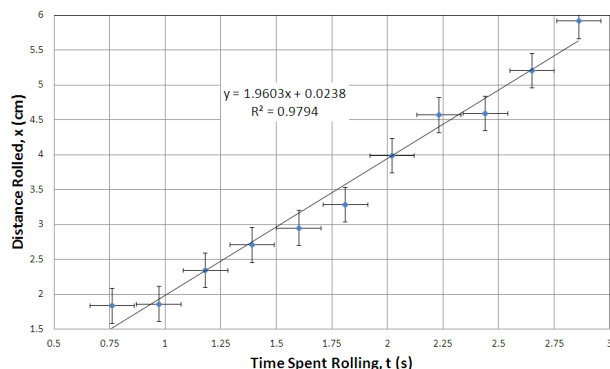
Figure 4: Example Error Bars

### 3.4.2   The Best-Fit Line

The data you plot in physics labs will almost always have a defining characteristic such as a straight line or parabolic trend. The best fit line is a representation of the visible trends in your raw data. Unless your data is very good your best fit line will rarely fall directly onto any of your data points. This is especially true of non-straight line plots.
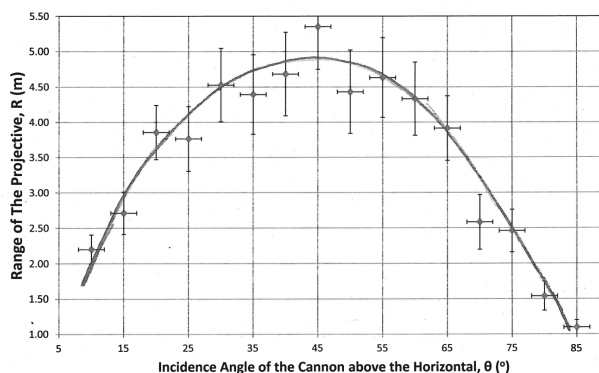
The two main methods of adding a best fit line to a plot are to draw it by hand or to fit the data with a computer package. To draw the line by hand simply draw a **smooth** curve through (but not necessarily touching) your data points. It should be within the error

Figure 5: Example Trend Lines

(a) Straight Line Computer Generated Fit

(b) Parabolic By Hand Fit

bars of all of the non-outlier points in your dataset. If you are fitting a straight line **USE A RULER**. Electronic fitting techniques will differ by software package used. If the lab manual or instructor does not specify a specific graphing program to use you may choose whatever program you are comfortable with as long as the plots look professional and follow all of the requirements outlined in this document. Ask your instructors for help if you are unfamiliar with computer graphing and analysis. The labs are a perfect opportunity to develop this skill. An example of each type of trend line is shown in Figure 5. Note: Make sure that your graphing program does not "connect the dots" between the data points. These are not best fit lines.

### 3.4.3 Uncertainty in Slopes and Arbitrary Points of Straight Lines

Quite often relative error expressions contain reference to the probable relative error in the slope $\frac{\Delta S}{S}$, of a graphical plot. You can determine the uncertainty in the slope using the "by hand three slope method" or by computer through a linear regression fitting method. Each method is described below.

**By Hand Method**

To determine the uncertainty in the slope without a computer you should use the three slope method. The tree slope method requires you to draw the best fit line, the maximum slope, and the minimum slope that reasonably fit the data. The relative uncertainty in the slope is then,

$$\frac{\Delta S}{S} = \frac{S_{\mathrm{max}} - S_{\mathrm{min}}}{2S} \tag{3.8}$$

where $S_{\mathrm{max}}$ and $S_{\mathrm{min}}$ are the most probable upper and lower limits of the slope and S is the best estimate of the slope. To establish the limits of the slope requires knowledge of the probable error limits in the measured co-ordinates (their uncertainties) which make up the plot. The probable limits of the slope are then set by the requirement that the lines of the slope maxima be drawn such as to not exceed the most probable error limits of all the legitimate (acceptable) points on the plot. That is, the maximum and minimum slope lines should be drawn so that no part of the line is outside the error bars of any datapoint unless that point is considered an outlier.

The graph below illustrates these points for the case of measurements to verify Hooke's law. Note the convention that the independent variable (force) is plotted on the x-axis and the dependent variable (displacement) is plotted on the y-axis. See Figure 6 for an example of this method.

To determine the uncertainty in points read from the best fit line subtract the value of the same point on the line of maximum slope from the value at the same point on the minimum slope and divide by two. Again see Figure 6 for an example of this method.

**By Computer Method**

When a best fit line is generated with a computer it will typically come with a fitting parameter which indicates how successful the fit was. In EXCEL this is the $R^2$ parameter available under the trend line properties. When adding the fit line to your plot also add the equation of the line and the fitting parameter and clearly label them as such. This is especially important for graphs with multiple data sets. The relative uncertainty in the slope is then taken as,

$$\frac{\Delta S}{S} = \left(1 - \sqrt{R^2}\right) \tag{3.9}$$

An example of this method is given as part of the example simple pendulum lab report at the end of this document. The relative uncertainty in any arbitrary point read from the best fit line can be taken as equal to the relative uncertainty in the slope.

## 3.5   Further Reading on Uncertainty

For those who wish to pursue further the topic of the treatment of experimental measurements the following books are suggested:

- Baird, D.C., Experimentation: An Introduction to Measurement Theory and Experimental Design, Prentice Hall, 1962.

- Beavington, P.R., Data Reduction and Error Analysis for the Physics Sciences, McGraw Hill, 1969.

- Young, H.D., Statistical Treatment of Experimental Data, McGraw Hill, 1962.

Figure 1: The distance a spring stretches versus the Force applied to stretch the spring to determine the spring constant
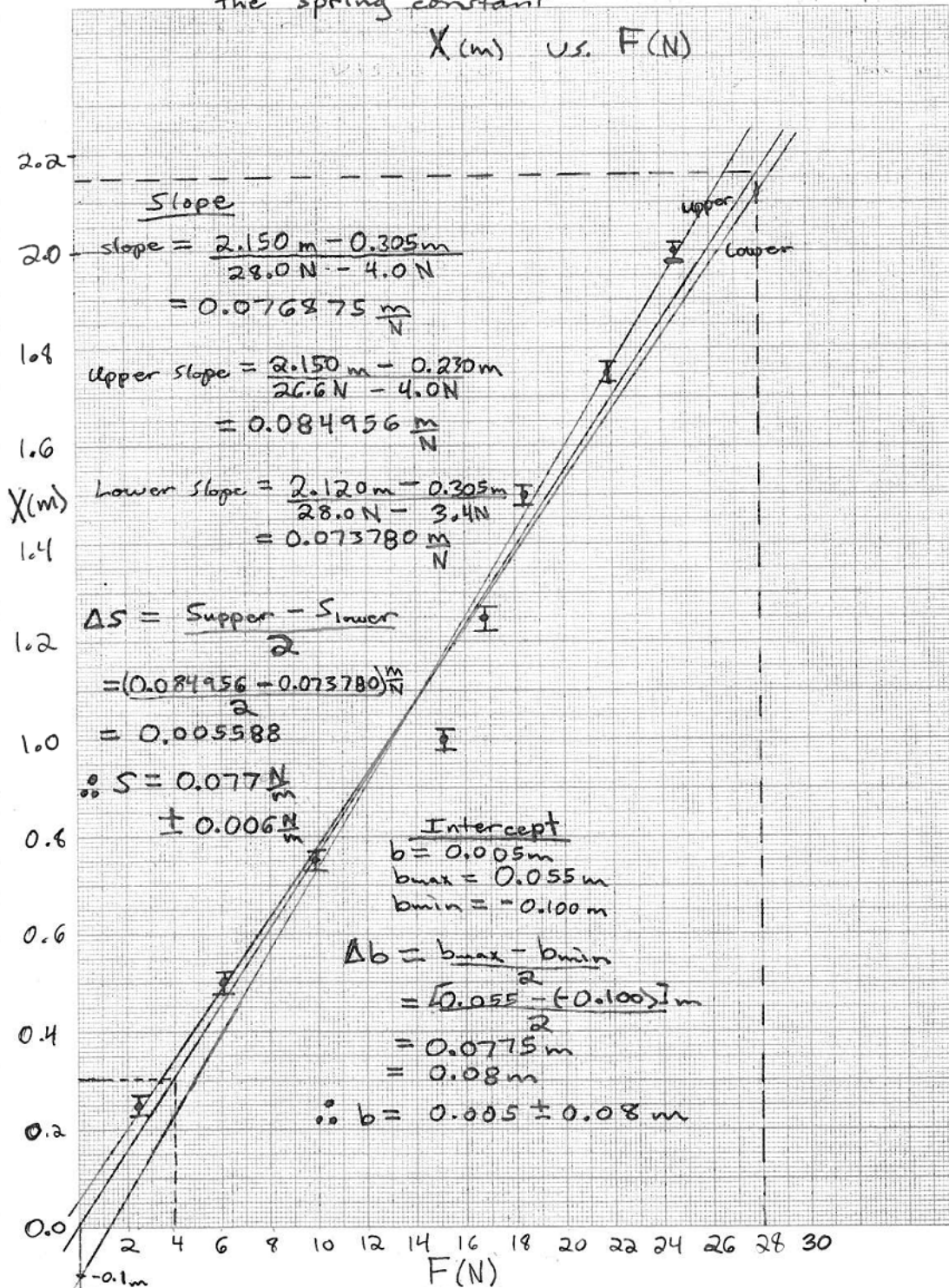
$$X_{(m)} \text{ vs. } F(N)$$



Slope

$$\text{slope} = \frac{2.150\,m - 0.305\,m}{28.0\,N - 4.0\,N}$$

$$= 0.076875 \frac{m}{N}$$

$$\text{Upper slope} = \frac{2.150\,m - 0.230\,m}{26.6\,N - 4.0\,N}$$

$$= 0.084956 \frac{m}{N}$$

$$\text{Lower slope} = \frac{2.120\,m - 0.305\,m}{28.0\,N - 3.4N}$$

$$= 0.073780 \frac{m}{N}$$

$$\Delta S = \frac{S_{upper} - S_{lower}}{2}$$

$$= \frac{(0.084956 - 0.073780)\frac{m}{N}}{2}$$

$$= 0.005588$$

$$\therefore S = 0.077 \frac{N}{m}$$

$$\pm 0.006 \frac{N}{m}$$

Intercept
$$b = 0.005\,m$$
$$b_{max} = 0.055\,m$$
$$b_{min} = -0.100\,m$$

$$\Delta b = \frac{b_{max} - b_{min}}{2}$$

$$= \frac{[0.055 - (-0.100)]\,m}{2}$$

$$= 0.0775\,m$$

$$= 0.08\,m$$

$$\therefore b = 0.005 \pm 0.08\,m$$

Figure 6: Three Slope Method

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# 4    Example Lab Report

What follows on this page is an example of what you might find in the lab manual to guide you through an experiment. On the following pages an example lab report that demonstrates many of the techniques discussed in this guide is presented for this simple experiment.

## Example Experiment: The Simple Pendulum

### Theory

A simple pendulum consists of a mass M hung from a fixed point P by a massless and infinitely thin string of length l, Figure 1.

The sum of the torque's on m as it swings about the fixed point will be,

$\sum \tau = I\alpha$

$-lmg\sin\theta = ml^2 \frac{d^2\theta}{dt^2}$

Using small angle approximation $\sin\theta = \theta$

$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$ This is the standard equation for simple harmonic motion with period,
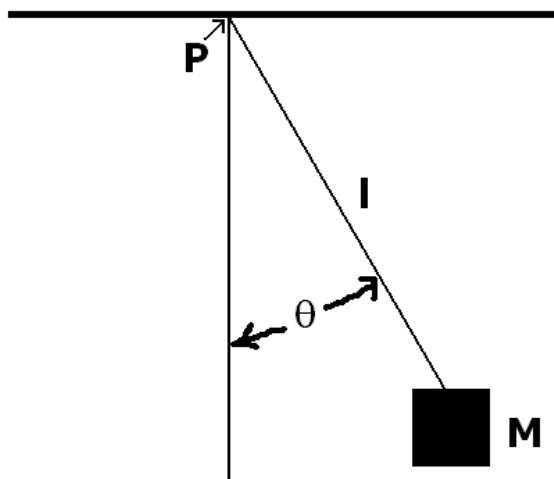
$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (4.1)$$



Figure 7: The Simple Pendulum

### Data Collection and Analysis

Using the equipment provided measure the period of oscillation for your simple pendulum system for several string lengths. To get a better result you should measure the time for several periods (at least 10) and divide by the number of periods you measured. Plot a graph to measure the gravitational acceleration of the Earth and compare with the accepted value of g = 9.81 $m/s^2$.

### Discussion

How well does the measured value of g agree with the experimental value? Explain any discrepancies.

Name: John Student
ID: 99999999
Partner: Jane Student (no relation)
Instructor: Mr. Instructor
Section: 001
Date: Feb 29, 2115

## Phys XXX: Example Experiment 1: The Simple Pendulum

### Purpose

The Purpose of this experiment is to verify the relationship between period and string length of a simple pendulum. As well we will verify the value of the gravitational acceleration of objects near the Earth's surface for our location on Earth.

### Results and Calculations

The time for the pendulum to complete 25 periods was measured at string lengths between 0.2m and 1.0m in steps of 0.2m. The results are summarized in Table 3 below. The uncertainty in the period measurements was chosen to be 0.5s even though the stopwatch measures to the hundredth of a second to account for human reaction times. The uncertainty in the string length was chosen to be 3mm to account for the difficulty of measuring the length as the string hung from the pendulum stand. The mass tended to sway back and forth if not held still but holding it tended to stretch the string.

By examining equation 4.1 and comparing it with the equation of a straight line $y = Sx + b$ we can see that a plot of $T^2$ versus $l$ should produce a straight line with slope $4\pi^2/g$ and no intercept. The values of $T^2$ for different string lengths are listed in Table 3.

Table 3: String Length, Period, Period Squared Measurements

| $l(m)$ | $T_{25}(s)$ | $T(s)$ | $T^2(s^2)$ | $\Delta T^2(s^2)$ |
| $\pm 0.03$m | $\pm 0.5$s | $\pm 0.02$s | | |
|---|---|---|---|---|
| 0.20 | 23.85 | 0.9540 | 0.910116 | 0.0382 |
| 0.40 | 33.08 | 1.3232 | 1.750858 | 0.0529 |
| 0.60 | 40.06 | 1.6024 | 2.567686 | 0.0641 |
| 0.80 | 45.79 | 1.8316 | 3.354759 | 0.0733 |
| 1.00 | 51.14 | 2.0456 | 4.184479 | 0.0818 |

Sample calculations for Table 3 using row 1,

$T = \frac{T_{25}}{25} = \frac{23.85s}{25} = 0.9540s$

$T^2 = (0.9540s)^2 = 0.910116s^2$

$\Delta T = T\sqrt{(\frac{\Delta T_{25}}{T_{25}})^2 + 0^2} = \frac{T_{25}}{25}\frac{\Delta T_{25}}{T_{25}} = \frac{\Delta T_{25}}{25} = \frac{0.5s}{25} = 0.02s$

$\Delta T^2 = T^2\frac{2\Delta T}{T} = 2T\Delta T = 2(0.9540s)(0.02s) = 0.0382s^2$

A plot of $T^2$ versus $l$ is attached as Figure 1 at the end of this report. It is clearly seen that the data fits a straight line very well as indicated by the $R^2$ fitting parameter as expected from equation 4.1. The slope of the EXCEL fit is read directly from the plot as $S = 4.076314 s^2/m$ and the uncertainty in the slope will be,

$$\Delta S = S(1 - \sqrt{R^2}) = 4.076314 s^2/m(1 - \sqrt{0.999891}) = 0.000222 s^2/m$$

As mentioned earlier in this section by examining equation 4.1 we see that a plot of $T^2$ versus $l$ will have a slope of,

$$S = \frac{4\pi^2}{g}$$

So solving for g,

$$g = \frac{4\pi^2}{S} = \frac{4\pi^2}{4.076314 s^2/m} = 9.684832 m/s^2$$

with uncertainty,

$$\Delta g = g\sqrt{\left(\frac{\Delta S}{S}\right)^2 + (0)^2} = 9.684832 m/s^2 \left(\frac{0.000222 s^2/m}{4.076314 s^2/m}\right) = 0.000527 m/s^2 = 0.0005 m/s^2$$

So finally, $g = 9.6848 \pm 0.0005 m/s^2$

$$\%Difference = \frac{|g_{\text{theory}} - g_{\text{measured}}|}{g_{\text{theory}}}100 = \frac{|9.81 m/s^2 - 9.6848 m/s^2|}{9.81 m/s^2}100 = 1.3\%$$

## Discussion

As mentioned previously our plot of period squared versus string length shows excellent agreement with the expected straight line behavior dictated by equation 4.1. As well our measured value of gravitational acceleration, g, is a mere 1.3 percent lower than the accepted value indicating good agreement. However, this 1.3 percent is outside the uncertainty limits calculated for g. The reasons for this are discussed below.

One of the assumptions of the theory is that the string is of fixed length. When performing the experiment it was noticed that the string could be stretched a small amount without much force applied. It is possible that the string stretched slightly at the bottom part of the curve it swings through while being less stretched at the top points of its motion because gravity will be pulling directly down along the string at the bottom portion. This would cause a decrease in the effective length of the string on average compared to the situation when originally measuring the length. which would be more pronounced for longer string lengths. This will lower the measured slope and thus increase the measured value of g. To counter this problem a rigid bar can be used to stop any deformation.

The above is provided as a sample discussion paragraph. In a real lab report a similar analysis would also be conducted using the following discrepancies:

1. The string is not an ideal string. That is, it is not massless and it's mass would have some effect on the moment of inertia.

2. The mass attached to the string is not a point mass exactly at the end of the string and should be considered as a cylinder for calculations of the moment of inertia.

3. Some of the motion of the pendulum might be in the 3rd dimension which was not considered in the derivation of the motion.

4. Anything else you can think of ...

**Conclusions**

This experiment has satisfied both of the goals we set out to achieve. The equation governing the period of a simple pendulum was verified by checking the straight line behavior of our period squared versus string length plot. The gravitational acceleration of the Earth was calculated to be 1.3% different from the accepted value which shows good agreement.
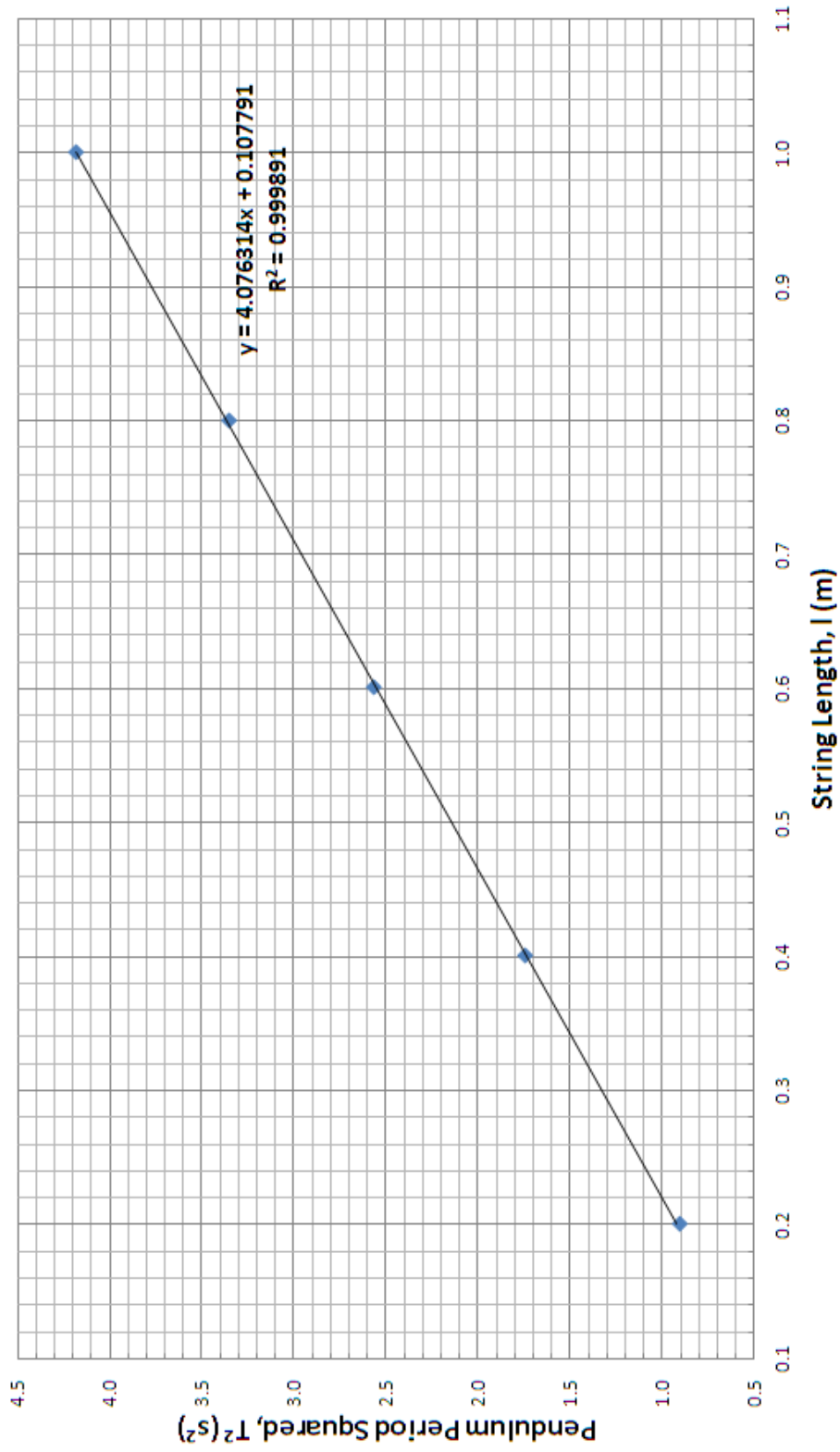
Figure 8: Period Squared vs. Pendulum Length

# 5 Special Calculations

This section describes some of the most common and most important calculations you will do on most of your physics labs.

## 5.1 The Percentage Difference Calculation

One of the easiest and most useful ways to compare two values is to calculate the difference between the values as a percentage of either one of the numbers or the average of the numbers. This provides a quantitative way for you to assess how close the numbers are. Percent difference is superior to just calculating the absolute difference between the numbers because it takes into account the order of magnitude of the values. For example, having a difference of $10^6 m$ would be quite small if you are dealing with lengths on the astronomical scale while it would be quite large for measuring distances on the atomic scale. Two methods for calculating percent difference are shown below.

Note: Percent difference is sometimes refered to as percent error. Since the term percent error can refer to either the percent uncertainty or the percent difference you should **never** use it to avoid confusing the reader.

**Method 1: When you have an accepted value**

If you are comparing your results to a given accepted value use the formula below.

$$\%Difference = \frac{|Accepted - Measured|}{Accepted}100 \tag{5.1}$$

Where Accepted is the accepted value you are comparing to and Measured is the final result you have calculated from your measurements and observations in the lab.

**Method 2: When you don't have an accepted value**

When there is no clear choice for an accepted value you can either choose whichever of the numbers you are most confident in as the reference value for use in equation 5.1 or use the equation below.

$$\%Difference = \frac{|Measurement1 - Measurement2|}{Average}100 \tag{5.2}$$

Where Measurement1 and Measurement2 are the results to be compared and Average is the average of the results calculated in the standard was (see equation 5.3).

## 5.2   The Average

Calculating averages are some of the most common and important calculations you will perform in your physics labs. Below are outlined two methods to calculate averages and their uncertainties.

### Method 1: The Normal Average

The normal average you are probably already familiar with is calculated by simply summing the value of each measurement and dividing by the number of measurements. To calculate an average of a set of n measurements, $x_i$, evaluate,

$$\overline{X} = \frac{\sum_i^n x_i}{n} = \frac{x_1 + x_2 + x_3 + x_4 + \ldots x_n}{n} \tag{5.3}$$

The uncertainty in the average is found by applying the normal uncertainty propagation rules for addition and division. Since the average calculation is so common the derivation of the uncertainty equation can usually be skipped. You must still show the equation and relevant calculation steps. When the uncertainty of each measurement is different the equation is,

$$\Delta\overline{X} = \frac{\sqrt{\sum_{i=1}^n \Delta x_i^2}}{n} = \frac{\sqrt{\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 + \Delta x_4^2 + \ldots \Delta x_n^2}}{n} \tag{5.4}$$

If the uncertainty on **all** of the measurements is equal then it is obvious that the above relation simplifies to,

$$\Delta\overline{X} = \frac{\Delta x_i}{\sqrt{n}} \tag{5.5}$$

### Method 2: The Weighted Average

Another method to calculate the average is by assigning each value a weight based off of its uncertainty and performing a weighted average calculation. If we have a set of measurements $x_i$ with uncertainty $\Delta x_i$ then the weighted average of these values is,

$$\overline{X} = \frac{\sum_{i=1}^n \frac{x_i}{\Delta x_i^2}}{\sum_{i=1}^n \frac{1}{\Delta x_i^2}} \tag{5.6}$$

It is clear from the above equation that the higher the uncertainty on a value the lower it's importance in the average. The uncertainty in the final average is then,

$$\Delta\overline{X} = \sqrt{\frac{1}{\sum_{i=1}^n \frac{1}{\Delta x_i^2}}} \tag{5.7}$$

It can be seen that if all of the uncertainty values used in the weighted average calculation are equal then the equations above simply reduce to those of the normal average case.

# 6   Useful Information

## 6.1   Use of the Vernier Caliper

Vernier scales are employed in many instruments, calipers, spectrometers, etc. All verniers have a main scale and an auxiliary or vernier scale. The calipers provided for most undergraduate physics experiments have vernier scale divisions that are 49/50 of the length of the main scale mm divisions and allow the main scale to be read to the nearest .02 mm.

The index mark on the vernier scale indicates the position to be read. From the main scale we see that this index mark lies between 0.8 and 0.9 cm in the example shown in figure 9.

The 4.2 line of the vernier is coincident with a line of the main scale. The next two decimal places are therefore read as .042. So the complete reading is 0.842 cm.

The reasoning is as follows: Each division on the vernier scale is shorter than each of the main scale divisions by 1/50 mm. In this example, the 4.2 mark on the vernier scale lines up best with a mark on the main scale. Since the 4.2 mark is the $21^{st}$ division from the index mark, the index mark must be replaced by $\frac{21}{50} = 0.42$ mm or 0.042 cm from the 0.8 mark.
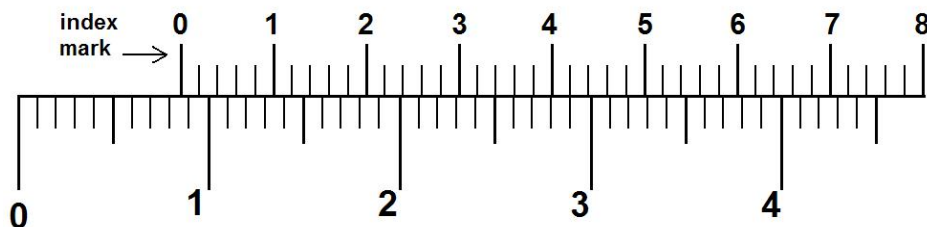


Figure 9: The Vernier Caliper

Since the smallest unit of precision is 0.002 cm the uncertainty in this reading is taken as 0.001 cm. This makes the final measurement 0.842 cm $\pm$ 0.001 cm.

## 6.2   Use of the Micrometer Caliper

The micrometer caliper shown in figure 10, commonly referred to as simply a micrometer, is an instrument for measuring small distances with an accuracy of about 0.001 mm or 1 micrometer.

To measure a distance with the micrometer, close the jaws of the micrometer gently on the object (undue pressure will deform the object). The upper part of the linear scale, A, on the shaft of the micrometer is graduated in mm and the  mm positions are indicated on the lower part. The fractional part of each mm is read on the circular scale B. The micrometer shaft must be turned through two complete revolutions to advance the shaft by 1 mm. The circular scale B on the shaft is divided into 50 divisions so that 1 division represents 0.01

mm. By estimating the fractional distance between these divisions the micrometer can be read to 0.001 mm. Thus, if you express your reading in mm, you must have three significant figures following the decimal point.

It is **important** to note that 0.500 mm must be added on to the reading of the circular scale whenever the shaft is more than halfway between two mm divisions on the linear scale A. (The purpose of the lower set of marks on scale A is to remind you of this!).

**Zero Error**. Due to normal wear a micrometer will, in general, not read exactly zero when the jaws are closed with nothing between them. (Assuming you are using the pressure normally used in taking a reading). All subsequent readings must be corrected by adding (or subtracting) the reading that the zero mark is behind (or ahead) of the reference line when the micrometer is closed. Note that the zero error must be also be read to the nearest 0.001 mm.
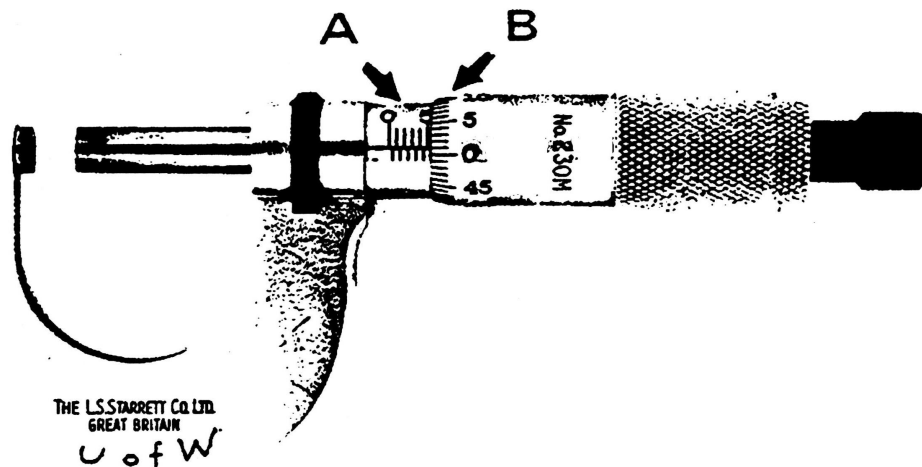


Figure 10: The Micrometer Caliper

The uncertainty of the micrometer is generally taken as ± 0.005 mm (half the smallest scale). As an example the reading in figure 10 would be 5mm from the large scale and 0.010mm from the small scale so that the final measurement is 5.010 mm ± 0.005mm.

## 6.3   Resistor Color Codes

All resistor values can be identified by the colored bands which typically surround the device. These bands always begin closer to one end of the resistor, and this is the end you begin reading from. Each color corresponds to a number from 0 to 9 and a multiplier from $10^0$ to $10^9$ as shown in table 4.

Table 4: Resistor Color Codes

| Color | Number | Multiplier |
|---|---|---|
| Black | 0 | $10^0$ |
| Brown | 1 | $10^1$ |
| Red | 2 | $10^2$ |
| Orange | 3 | $10^3$ |
| Yellow | 4 | $10^4$ |
| Green | 5 | $10^5$ |
| Blue | 6 | $10^6$ |
| Violet | 7 | $10^7$ |
| Grey | 8 | $10^8$ |
| White | 9 | $10^9$ |

The first and second bands represent the first two digits of the resistance, the third band is the multiplier.

So, for example a band structure of Brown, Black, Red, yields:

10 x 100 = 1000 or 1.0 k$\Omega$

Typically, each resistor has an associated tolerance. This information is given by the fourth band in the sequence. A gold band represents $\pm 5\%$ a silver band represents $\pm 10\%$ and the absence of a fourth band represents a $\pm 20\%$ tolerance. Thus the sequence Orange, Orange, Yellow, Silver represents:

33 x $10^4 \Omega \pm 10\%$ or 330 k$\Omega \pm$ 33k$\Omega$