The Franck-Hertz Experiment

Andy Chmilenko, 20310799
Instructor: Tan Dinh
Section 1
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I. PURPOSE

The purpose of this experiment is to study the quantization of energy levels in Hg atoms in a low pressure vapour using electrons, specifically using a technique known as electron energy loss spectroscopy (EELS) to determine the energy for the lowest excited state for Hg and relate this to that of the ground state and energy level diagram for Hg.

II. ANALYSIS

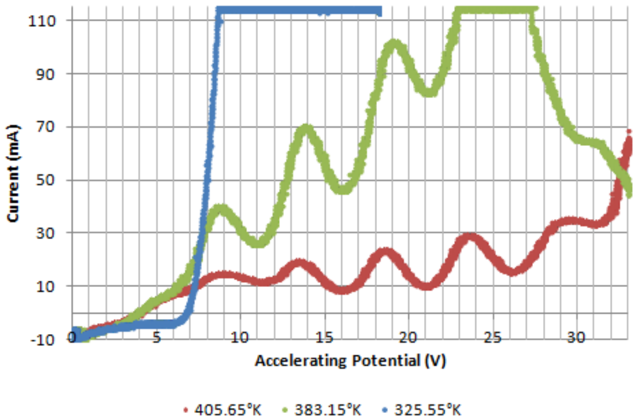


FIG. 1: Measured current versus the electron accelerating potential of the Franck-Hertz unit operating on a Franck-Hertz tube filled with Hg Vapour at 132.5°C, 110.0°C, and 52.4°C, in red, green, and blue respectively. Note the Frank-Hertz signature is lost at low temperatures

Temperature of F-H tube (°C)	Maxima (V) ± 0.05V	δ Adjacent Maxima (V) ± 0.07V	Minima (V) ± 0.05V	δ Adjacent Minima (V) ± 0.07V
132.5	4.4, 8.8, 13.4, 18.5, 23.6 , 29.2	4.4, 4.6, 5.1, 5.1, 5.6	6.5, 11.2, 15.9, 21.0, 26.1, 31.1	4.7, 4.7, 5.1, 5.1, 5.0
110.0	4.4, 8.6, 13.8, 19.0	4.2, 5.2, 5.2	6.2, 11.0, 16.0, 21.1	4.8, 5.0, 5.1

TABLE I: Approximate local minima and maxima for the curves created by the data points in Fig 1, and the differences between adjacent local minima and maxima.

Sample Calculations for δ Adjacent Maxima using Table I row 1, between the two points, 4.4 V and 8.8 V

$$\Delta V = V_2 - V_1$$
$$\Delta V = 8.8 - 4.4$$
$$\Delta V = 4.4V$$

Sample Calculations for average change in local maxima using Table I, column 3

$$\overline{V} = \frac{\sum_i^n \delta_{maxima_i}}{n}$$
$$\overline{V} = \frac{4.4+4.6+5.1+5.1+5.6+4.2+5.2+5.2}{8}$$
$$\overline{V} = 4.925V$$

Sample Calculations for uncertainty in $\overline{V}$, assuming an error of measurement of 0.05V

$$\Delta\overline{V} = \frac{\sqrt{\sum_i^n \delta_{m_i}^2}}{n}$$
$$\Delta\overline{V} = \frac{\sqrt{n \cdot \sqrt{\delta_{m1}^2 + \delta_{m2}^2}^2}}{n}$$
$$\Delta\overline{V} = \frac{\sqrt{8 \cdot (0.05^2 + 0.05^2)}}{8}$$
$$\Delta\overline{V} = \frac{0.2}{8}$$
$$\Delta\overline{V} = 0.025V$$

By analysing the data in the curves made by the data points as seen in Fig.1, several maxima and minima can be found as seen in Table I, and the value between adjacent local minima and maxima can be calculated, as well as the uncertainty in this calculated value assuming we take half of the least significant digit to be 0.05V. The average potential change between local maxima was found to be when the accelerating potential 4.9V $\pm$ 2.5$\times10^{-2}$V, meaning the electrons equivalently had that amount of kinetic energy in eV, and similarly the average potential change between local minima was found to be 4.9V $\pm$ 2.5$\times10^{-2}$V as well. Measuring the difference between local maxima and minima yield the same result in finding the excitation energy of Hg, which in this case is approximately 4.9eV $\pm$ 2.5$\times10^{-2}$eV.

In this case, it's the $6s6p^3P_{0,1,2}$ energy levels that are getting excited in collisions with electrons, since these 3 levels are close together around the 4.9eV line.

$$\varepsilon = w_0 = \frac{1}{2} m_e v^2 \tag{1}$$

The first maxima was found to be around 4.4eV which is the electron's work function on an Hg atom, the minimum amount of energy needed for an electron to ionize Hg. The work function for Hg is likely slightly higher than measured, as electrons get some energy from the thermionic emission from the filament in the F-H tube, but this energy is negligible relative energy to the accelerating potential from the anode, so 4.4eV is quite close to the actual work function of Hg. Using Eq.1 we can calculate the speed of electrons at this kinetic energy.

Sample Calculations for speed of an electron with a kinetic energy of 4.4eV

$$w_0 = \frac{1}{2}m_e v^2$$
$$e \times 4.4 = \frac{1}{2}9.11 \times 10^{-31} v^2$$
$$v^2 = \frac{2e\times 4.4}{9.11\times 10^{-31}}$$
$$v = \sqrt{1.55 \times 10^{12}}$$
$$v = 1.24 \times 10^6 \frac{m}{s}$$

Since the kinetic energy of the electron isn't too high and the velocity is small relative to the speed of light, we don't need to really account for relativistic effects, so the velocity of the electron should be quite close to $1.24{\times}10^6 \frac{m}{s}$.

$$PV = NkT \tag{2}$$

$$MFP = (\sigma n)^{-1} = (\pi r^2 n)^{-1} \tag{3}$$

Sample Calculations for number density of Hg at 132.5°C (405.65°K) with a Vapour pressure of 181.87 Pa

$$PV = NkT$$
$$\frac{N}{V} = n = \frac{P}{kT}$$
$$n = \frac{181.87}{1.38\times 10^{-23}\cdot 405.65}$$
$$n = 3.25 \times 10^{22} \frac{atoms}{m^{-3}}$$

Sample Calculations for the mean free path of electrons in an Hg vapour that have an atomic radius of approximately 150pm with a number density of 3.25 × 10^{22} $\frac{atoms}{m^{-3}}$

$$MFP = (\pi r^2 n)^{-1}$$
$$MFP = (\pi(1.5 \times 10^{-10})^2 \cdot 3.25 \times 10^{22})^{-1}$$
$$MFP = (\pi(1.5 \times 10^{-10})^2 \cdot 3.25 \times 10^{22})^{-1}$$
$$MFP = 4.35 \times 10^{-4} \frac{m}{collision} = 4.35 \times 10^{-1} \frac{mm}{collision}$$

Sample Calculations collision rate of electrons travelling with a velocity of 1.24×10^6 $\frac{m}{s}$, and a calculated MFP of 4.35 × 10^{-4} m

$$Rate = \frac{v}{MFP}$$
$$Rate = \frac{1.24\times 10^6}{4.35\times 10^{-4}}$$
$$Rate = 2.85 \times 10^9 \frac{collision}{sec}$$

Sample Calculations uncertainty in vapour pressure, ρ, using row 1 from Table III

$$\Delta\rho = \rho \cdot \sqrt{(\frac{\Delta T}{T})^2 + 0.01^2}$$
$$\Delta\rho = 181.87 \cdot \sqrt{(\frac{2+132.5\cdot 0.02}{132.5})^2 + 0.01^2}$$
$$\Delta\rho = 181.87 \cdot \sqrt{0.035^2 + 0.01^2}$$
$$\Delta\rho = 181.87 \cdot 4\%$$
$$\Delta\rho = 7.3 Pa$$

Sample Calculations uncertainty in number density, n, using row 1 from Table III

$$\Delta n = n \cdot \sqrt{(\frac{\Delta T}{T})^2 + (\frac{\Delta P}{P})^2}$$
$$\Delta n = 3.25 \times 10^{22} \cdot \sqrt{0.035^2 + (\frac{\Delta 7.3}{181.87})^2}$$
$$\Delta n = 3.25 \times 10^{22} \cdot \sqrt{0.035^2 + 0.040^2}$$
$$\Delta n = 3.25 \times 10^{22} \cdot 0.053$$
$$\Delta n = 1.7 \times 10^{21}$$

Sample Calculations uncertainty in mean free path, MPF, using row 1 from Table III

$$\Delta MPF = MPF \cdot \frac{\Delta n}{n}$$
$$\Delta MPF = 4.35 \times 10^{-1} \cdot \frac{1.7\times 10^{21}}{3.25\times 10^{22}}$$
$$\Delta MPF = 4.35 \times 10^{-1} \cdot 0.052$$
$$\Delta MPF = 2.3 \times 10^{-2}$$

Sample Calculations uncertainty in collision, using row 1 from Table III

$$\Delta Rate = Rate \cdot \sqrt{(\frac{\Delta v}{v})^2 + (\frac{\Delta MPF}{MPF})^2}$$
$$\Delta Rate = Rate \cdot \sqrt{(\frac{1}{2}\frac{\Delta K_e}{K_e})^2 + (\frac{\Delta MPF}{MPF})^2}$$
$$\Delta Rate = 2.85 \times 10^9 \cdot \sqrt{(\frac{1}{2}\frac{0.1}{4.4})^2 + (\frac{2.3\times 10^{-2}}{4.35\times 10^{-1}})^2}$$
$$\Delta Rate = 2.85 \times 10^9 \cdot \sqrt{0.0114^2 + 0.0529^2}$$
$$\Delta Rate = 2.85 \times 10^9 \cdot 0.054$$
$$\Delta Rate = 1.5 \times 10^8$$

Temperature of F-H tube (°C) ±(2°C+2%)	Vapour Pressure (Pa)	Number Density n $(\frac{atoms}{m^{-3}})$	Mean Free Path $(\frac{mm}{collision})$	Collision Rate $(\frac{collision}{sec})$
132.5	181.87	3.25×10^{22}	4.35×10^{-1}	2.85×10^{9}
110.0	62.09	1.17×10^{22}	1.21	1.02×10^{9}
52.4	2.12	4.72×10^{20}	30.0	4.13×10^{7}

TABLE II: Approximate local minima and maxima for the curves created by the data points in Fig 1, and the differences between adjacent local minima and maxima.

Temperature of F-H tube (°C) ±(2°C+2%)	Δ Vapour Pressure (Pa)	Δ Number Density n $(\frac{atoms}{m^{-3}})$	Δ Mean Free Path $(\frac{mm}{collision})$	Δ Collision Rate $(\frac{collision}{sec})$
132.5	7.3	1.7×10^{21}	2.3×10^{-2}	1.5×10^{8}
110.0	2.5	6.5×10^{20}	6.7×10^{-2}	5.8×10^{7}
52.4	1.3×10^{-1}	4.0×10^{19}	2.5	3.5×10^{6}

TABLE III: Calculated uncertainty from corresponding values in Table II

Using the table of Vapour pressures at specific temperatures as outlined in the CRC handbook the Vapour pressures were found and tabulated into Table II. Using Eq.2 the number density of the Hg in the F-H tube was also calculated, as well as the mean free path with Eq.3.

The calculated values and results are in line with the dimensions of the tube, with higher temperatures, like the 132.5°C and 110°C cases, where there is more Hg atoms, the MFP is higher and consequently we see a higher signal on the collector electrode. However for the low temperature situation at 52.4°C where the MFP was 30mm, which was much larger than the distance to the collector electrode the signal disappeared since there were little to no collisions between electrons and Hg atoms in the tube. The MFP isn't constant with different temperatures, as the vapour density is dependent on the temperature of the tube. The primary uncertainty in this calculation is from the temperature reading, as the pressure is also dependent on the temperature. The Protek TM-1300k Thermometer is rated to having an accuracy of 2°C + 2% within the range of temperatures the F-H tube was at, and the CRC has a 1% uncertainty with the vapour pressures of mercury.

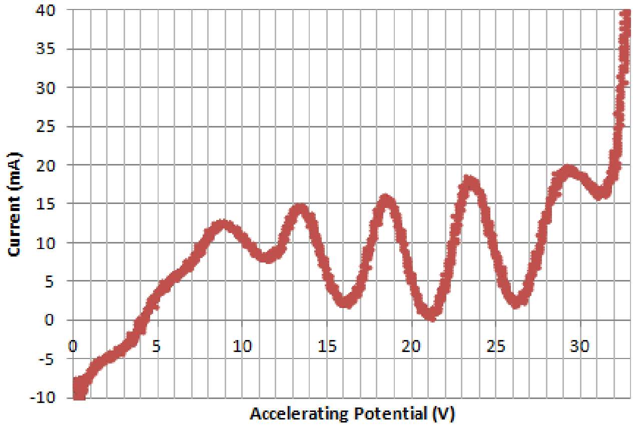


FIG. 2: Measured current versus the accelerating potential of the Franck-Hertz unit operating on a Franck-Hertz tube filled with Hg Vapour at 132.5°C with the subtracted background current.

By subtracting the background current where, according to the Childs-Langmuir law, I $\propto$ $V^{\frac{3}{2}}$, we can get a better reading for maxima on our data set, as seen in Fig.2. No evidence in this instance can be seen for excitations for more than one Hg energy level, perhaps at higher temperatures electron energies the other excitation energies may be more evident.

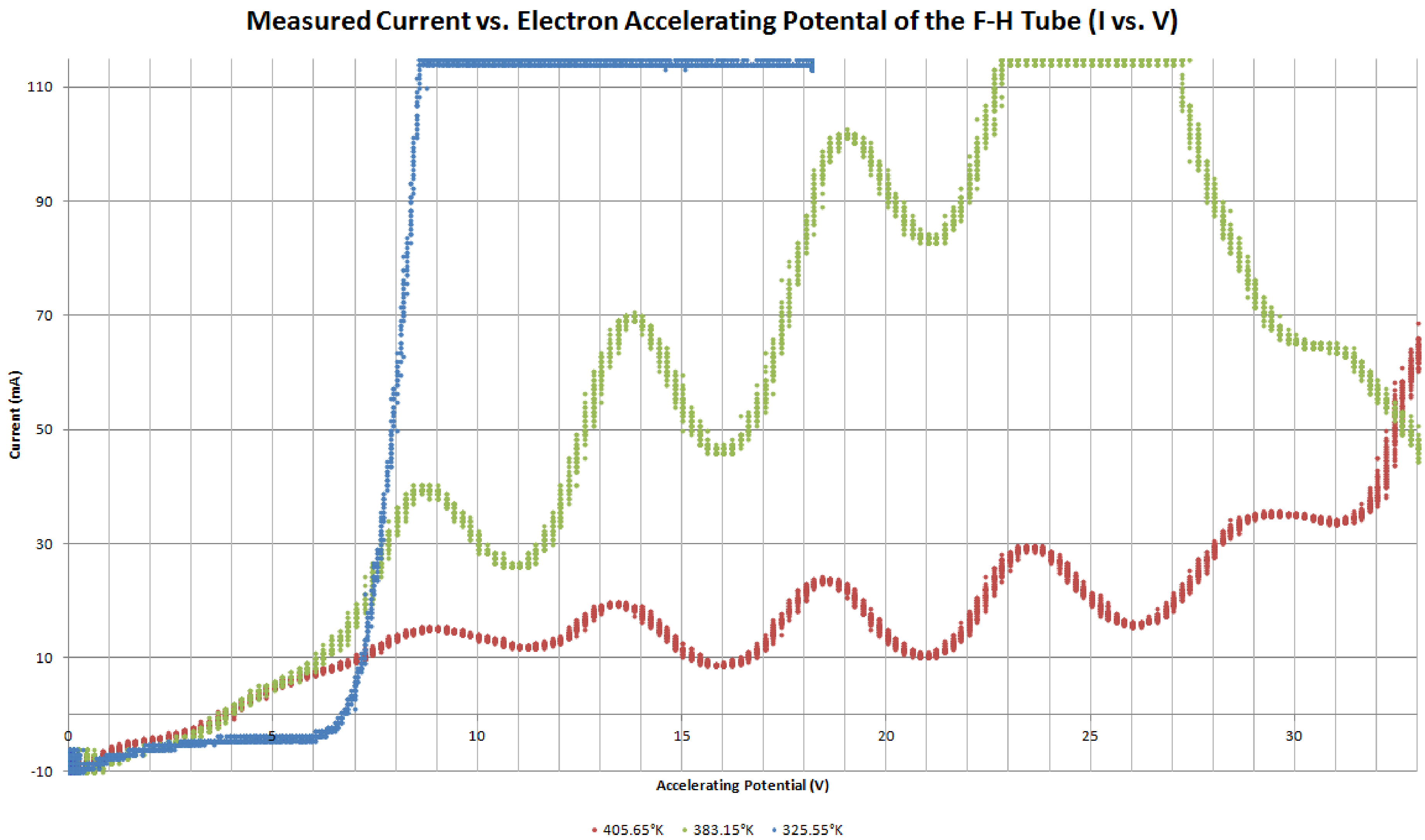
Most of the error incurred in this experiment was due the temperature readings and the thermometer. The thermometer is much more apt in getting an ambient temperature of the inside of the casing holding the F-H tube, some of the results and calculations could be made more accurate by being able to measure the F-H tube's temperature more directly.

III. CONCLUSION

The experiment confirmed the quantization of energy levels in Hg atoms in a low pressure vapour using electron energy loss spectroscopy, and the lowest level energy state of Hg was also determined to be 4.9eV $\pm2.5\times10^{-}2$eV. The work function, w_0 was also measured to be around the 4.4eV energy, with possibly a little bit extra due to the energy the electron gets from the thermionic emission from the filament.

The mean free paths were also calculated for the data collected at the 3 different temperatures. The calculations for the two high temperature cases at 132.5°C and 110°C were of 4.35×10^{-1} mm and 1.21mm respectively. It was also at these temperatures which we saw the Franck-Hertz signature and this result makes sense with respect to the dimensions of the tube, since the total distance the electron travels >> MFP of the electron. However for the 52.4°C, the temperature at which the Franck-Hertz signature wasn't visible, the MFP was calculated to be on 30.0mm which is much larger than the total distance it travels before being collected in the collecting electrode which was also in agreement.

A better reading of the measurements was also made by applying the Childs-Langmuir law, the relation that I $\propto$ $V^{\frac{3}{2}}$, we were able to observe the minima and maxima better.



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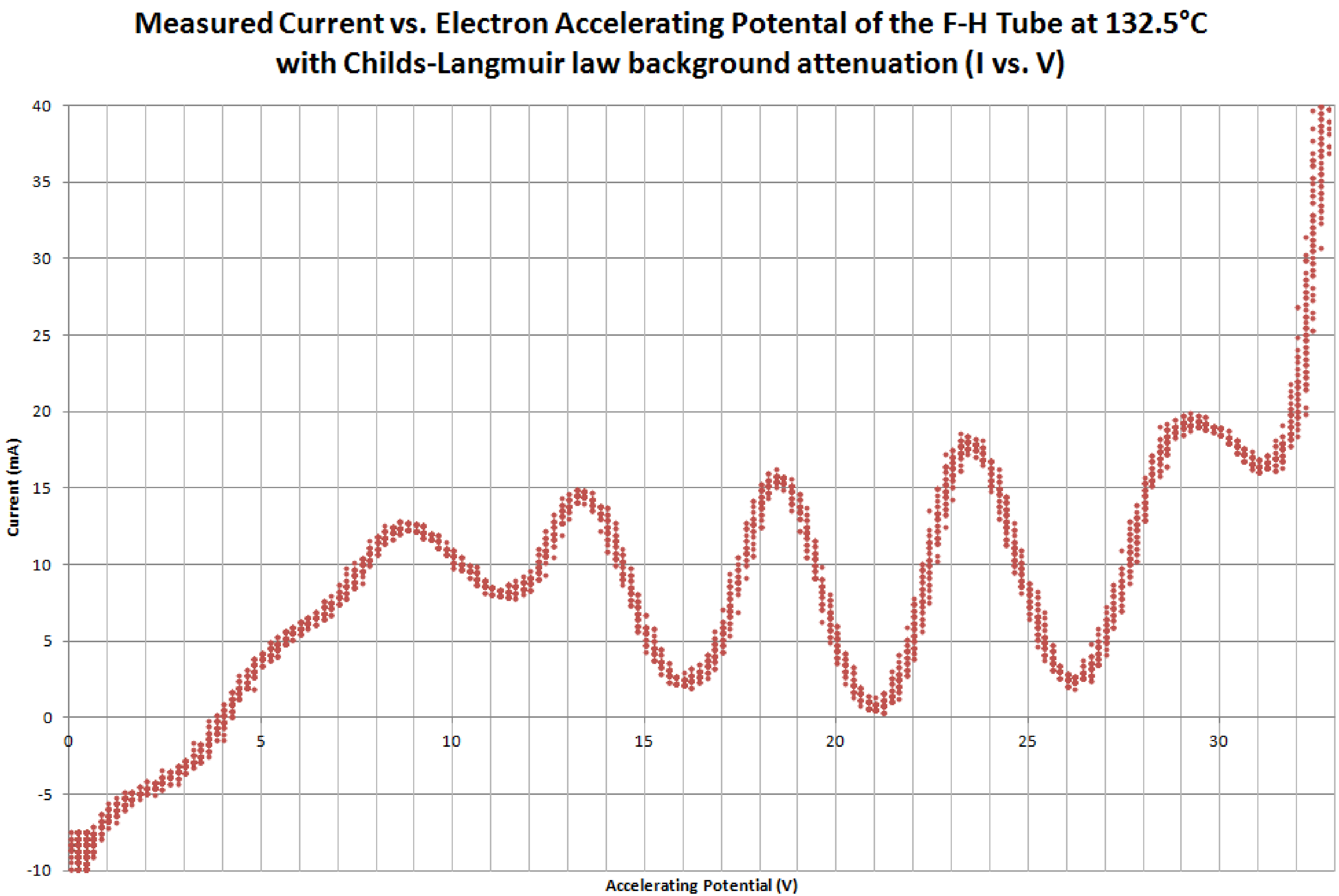

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