Thermal Radiation: The Stefan-Boltzmann Law

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Section 1
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I. PURPOSE

The purpose of this experiment is to verify the Stefan-Boltzmann Law, investigate aspects of blackbody radiation such as radiation rates from different surfaces and measure their emissivity (ε), investigating the absorption and transmission of thermal radiation, as well as verify the inverse square law with respect to the radiative power of our Stefan-Boltzmann lamp.

II. ANALYSIS

A. Radiation rates from different Surfaces

Cube No.	Thermistor Resistance (Ω)	Temperature T (°K)	Sensor reading (mV) Black	White	Dull Al	Polished Al
9	4,692	378.6	17.4	16.0	2.6	0.9
8	5,500	373.2	16.5	16.2	2.9	1.1
6	13,910	345.6	8.9	9.0	2.6	0.9
5	21,430	334.5	6.4	6.3	1.4	0.6

TABLE I: Thermistor measurements along with the calculated Temperatures for several Leslie cubes, as well as the sensor measurements from the Thermopile for all 4 sides of the cubes.

Sample calculations for T using row 1 of Table I

Using values from Table X in the Appendix, for a Thermistor Resistance of 4692 Ω, assuming a linear relationship between two adjacent resistances corresponding to a difference in 1 degree, using 4760.3 Ω → 105 °C and 4615.1 Ω → 106 °C we can correlate the difference in resistance with the difference in 1 degree.

$$T = 105°C + \%_{Difference} \cdot 1°C + 273.15°C$$
$$T = 105°C + \frac{4760.3-4692}{4760.3-4615.1} \cdot 1°C + 273.15°C$$
$$T = 378.6$$

The distribution function as seen in Eq.1 for the wavelength dependence of emission of thermal radiation by a blackbody at temperature T using Planck's theory of quantization.

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 \cdot e^{\frac{hc}{\lambda kT}}} \quad (1)$$

and integrating this over all wavelengths the total power is simply defined as Eq.2 and including a term ε, the emissivity, where an emissivity of 1 is a perfect blackbody, and $0 < \varepsilon < 1$ is an imperfect or non-blackbody.

$$P = \varepsilon\sigma T^4 \quad (2)$$

Using the thermopile, several surfaces of different temperatures were measured, and the data was compiled into Table I. Using this data we can calculate the relative emissivity of each face for all the Leslie cubes assuming that the black face of the cube has $\varepsilon = 1$, these values are compiled in Table II.

Cube No.	Emissivity ε Black	White	Dull Al	Polished Al
9	1	0.92	0.15	0.05
8	1	0.98	0.18	0.07
6	1	1.01	0.29	0.10
5	1	0.98	0.22	0.09

TABLE II: Relative emissivity for all faces of Leslie cubes, assuming the black face has an emissivity of 1

Sample calculations for White emissivity using row 1 of Table I, and assuming black emissivity is 1

$$\varepsilon_{white} = \frac{White_{emission}}{Black_{emission}}$$
$$\varepsilon_{white} = \frac{16.0}{17.4}$$
$$\varepsilon_{white} = 0.92$$

Emissivity in this case is independent of temperature, as the temperature decreases the emissivity stays more or less the same as seen in table II with standard deviation values in Table III. The dull and polished aluminium have some slightly varying emissivity values as the percent deviation values are around 28%, but this isn't too significant relative to radiative output compared to the black surface. The data show that better absorbers are better emitters, and poorer absorbers (better reflectors) are poorer emitters like in the case of the aluminium and even more extreme the polished aluminium compared to the black or white surfaces.

Surface	Mean Emissivity $\overline{\varepsilon}$	Percent Deviation
Black	1	0
White	0.97	4%
Dull Al	0.21	29%
Polish Al	0.08	28%

TABLE III: Mean emissivity and relative percent deviations from the mean for all the surfaces measured in Table II

Sample Calculations Averaging White Emissivity ε in Table II using column 3

$$\overline{\varepsilon} = \frac{\sum_i^n \varepsilon_i}{n}$$
$$\overline{\varepsilon} = \frac{0.92+0.98+1.01+0.98}{4}$$
$$\overline{\varepsilon} = 0.97$$

Sample Calculations for White standard deviation σ_ε in Table II using column 3 and a mean of 0.97

$$\sigma_\varepsilon = \sqrt{\frac{1}{n-1}\sum_i^n(\varepsilon_i - \overline{\varepsilon})^2}$$
$$\sigma_\varepsilon = \sqrt{\frac{1}{4-1}[(0.92-0.97)^2+(0.98-0.97)^2+ (1.01-0.97)^2+(0.98-0.97)^2]}$$
$$\sigma_\varepsilon = \sqrt{\frac{1}{3}0.0043}$$
$$\sigma_\varepsilon = 3.8\times10^{-2}$$

Sample Calculations for White percent deviation using a σ_ε of 3.8 $\times$ 10^{-2} and mean emissivity $\overline{\varepsilon}$ of 0.97 from Table III

$$\%_{deviation} = \frac{\sigma_\varepsilon}{\overline{\varepsilon}}\times 100$$
$$\%_{deviation} = \frac{3.8\times10^-2}{0.97}\times 100$$
$$\%_{deviation} = 4\%$$

B. Absorption and transmission of thermal radiation

Cube No.	I_0 (mV)	I_{glass} (mV)	$\frac{I_{glass}}{I_0}$
9	16.3	0.0	0
9	13.8	0.0	0

TABLE IV: Intensity of thermal radiation of the black surface measured through air and through a pane of glass

Sample Calculations for $\frac{I_{glass}}{I_0}$ using Table IV row 1

$$\frac{I_{glass}}{I_0} = \frac{0.0}{16.3} = 0$$

In this case we didn't have a strong enough signal to have any transmission through the glass, but we concluded that glass blocks most of the infrared radiation incident on it, as seen in Table IV. In this case, it doesn't seem like I_{glass} is dependent on I_0.

C. The Stefan Boltzmann law at low temperatures

The thermopile sensor can also be thought of as source of thermal radiation since it operates at room temperature. So the net power read by the sensor is the difference from the source of radiation it is measuring and the radiation from the detector itself since it radiates away some of the energy it is trying to measure. This is given by Eq.3

$$P_{net} = P_{rad} - P_{det} = \varepsilon\sigma T^4 - \varepsilon_{det}\sigma T_{det}^4 \tag{3}$$

We can try assuming that the P_{det} can be neglected in this experiment because it is low and doesn't account for much difference. Assuming $\epsilon = \epsilon_{det}$ we can rearrange Eq.3 as follows.

$$P_{rad} = \varepsilon\sigma T^4 - \varepsilon_{det}\sigma T_{det}^4$$
$$P_{rad} = \varepsilon\sigma(T^4 - T_{det}^4) = \varepsilon\sigma(T_k^4 - T_{rm}^4)$$

$$\frac{P_{rad}}{(T_k^4 - T_{rm}^4)} = \varepsilon\sigma = Const \tag{4}$$

Since our sensor measurement is directly proportional to the power of the radiation, we can graph P_{rad} versus (T_K^4 - $T_r m^4$) to confirm weather the Stefan- Boltzmann equation is correct.

Cube No.	Thermistor Resistance (Ω)	Temperature T (°K)	Sensor reading for Black (mV)
9	4,692	378.6	17.4
8	5,500	373.2	16.5
7	8,160	361.4	10.3
6	13,910	345.6	8.9
5	21,430	334.5	6.4
4	24,120	331.5	5.8
3	54,820	311.5	2.0
2	109,730	296.2	0.3
1	110,320	296.1	0.3
$Cube_{RT}$	112,220	295.7	0.3

TABLE V: Thermistor measurements along with the calculated Temperatures for several Leslie cubes, as well as the sensor measurements from the Thermopile for the black side of the cube.

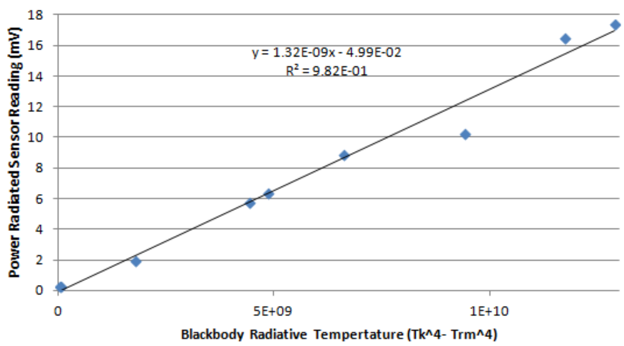

FIG. 1: Thermopile sensor measurements versus the temperature difference between the sample and the room.

As seen in Fig.1, we can see that this line fits the data very well, with one outlier data point with row 3 of Table V, but that withstanding, this proves the relationship of power and temperature in the Stefan-Boltzmann equation even at low temperatures.

D. Inverse Square Law

Distance (m)	Sensor Reading (mV)
2.2	0
10	0
20	0
30	0
40	0
50	0
60	0
70	0
80	0
90	0
100	0

TABLE VI: Thermistor measurements of the Stefan-Boltzmann lamp while it was off, for the average ambient measurements of the lab.

The average measurements of the lamp from column 2 of Table VI was calculated to be 0 mV, there was not enough infrared radiation in the lab to produce any significant measurements to the sensor.

Distance (m)	Sensor Reading (mV)
2.5	126.1
5	42.4
10	12.7
15	6.1
17	4.8
20	3.5
25	2.3
30	1.6
40	0.8
50	0.5
60	0.3
70	0.2
80	0.0
100	0.0

TABLE VII: Thermistor measurements of the Stefan-Boltzmann lamp running at a voltage of 9.99 V, at various distances.

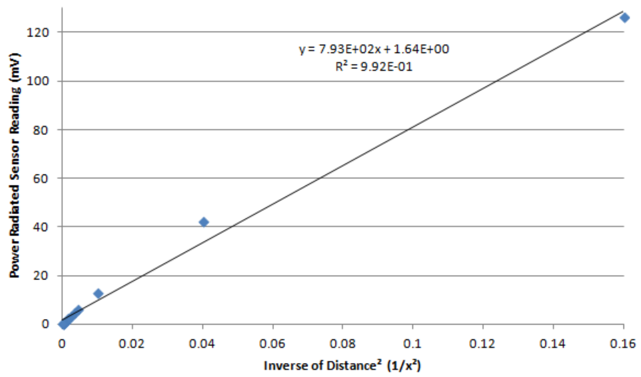

FIG. 2: Thermopile sensor measurements versus the inverse square of the distance from the Stefan-Boltzmann lamp

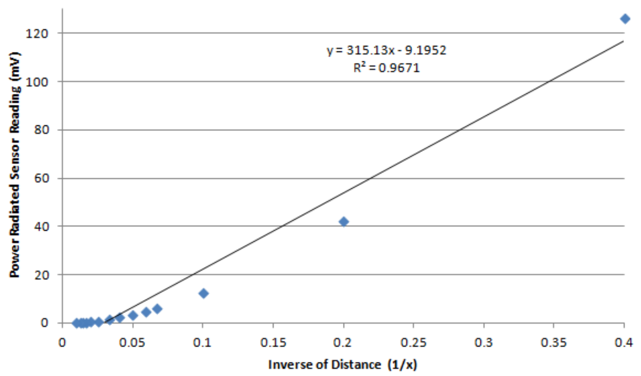

FIG. 3: Thermopile sensor measurements versus the inverse of the distance from the Stefan-Boltzmann lamp

The radiative power of the lamp does scale with the inverse square law as seen in Fig.2, however there is a deviation at the closer distances, as the power seems to fall off uncharacteristically. This deviates from the ideal result probably because of the shape of the filament of the Stefan-Boltzmann lamp not being a point source, as you get farther and farther away the point source approximation gets better.

One way the experiment could be changed is to have a much brighter, or conversely, much more sensitive thermopile such that the sensor can be put at a distance where it cannot resolve the size of the light. Another way could be to pass the light though a pin hole to approximate a really small source of light.

E. The Stefan-Boltzmann Law

As previously shown in Table V, the resistance of the Leslie cube at room temperature was measured to be 112.22 kΩ, T_{ref} was calculated to be 295.7 °K. R_{ref}, the reference resistance of the Stefan-Boltzmann lamp was calculated to be 0.3 Ω.

Lamp Voltage V_{lamp} (V)	Lamp Current I_{lamp} (A)	R_T (Ω)	$\frac{R_T}{R_{ref}}$	Temperature T (°K)
1.115	0.872	1.279	4.26	972
2.000	1.090	1.835	6.12	1320
3.001	1.308	2.294	7.65	1590
4.000	1.508	2.652	8.84	1800
5.001	1.687	2.964	9.88	1970
6.001	1.8563	3.233	10.78	2120
7.00	2.010	3.482	11.61	2260
8.00	2.155	3.712	12.37	2390
9.00	2.285	3.939	13.13	2510
10.00	2.410	4.149	13.83	2620
11.00	2.534	4.341	14.47	2710
12.00	2.647	4.533	15.11	2820

TABLE VIII: Stefan-Boltzmann lamp running at various voltages, with their associated current measurements, resistance calculations, and temperature calculations. Temperature was calculated with the same method as in Table I, except using Table XI in the Appendix as the lookup table for resistance to temperature values for the lamp.

Sample calculations for R_T using row 1 of Table VIII

$$R_T = \frac{Vlamp}{Ilamp}$$
$$R_T = \frac{1.115V}{0.872A}$$
$$R_T = 1.279\Omega$$

Sample calculations for $\frac{R_T}{R_{ref}}$ using row 1 of Table VIII with R_{ref} of 0.3

$$\frac{R_T}{R_{ref}} = \frac{1.279}{0.3}$$
$$\frac{R_T}{R_{ref}} = 4.26$$

Sensor Reading (mV)	Temperature T^4 (°K^4)
0.2	8.93×10^{11}
1.2	3.04×10^{12}
3.1	6.39×10^{12}
5.7	1.05×10^{13}
8.8	1.51×10^{13}
12.3	2.02×10^{13}
16.9	2.61×10^{13}
21.6	3.26×10^{13}
26.4	3.97×10^{13}
31.5	4.71×10^{13}
36.3	5.39×10^{13}
41.8	6.32×10^{13}

TABLE IX: Thermopile sensor measurements of the Stefan-Boltzmann lamp and its Temperature T^4.

As it can be seen in Fig.4, a power-regression shows that our data fits an $x^{4.95}$ relationship which is close to our predictions according to the Stefan-Boltzmann Law (Eq.2) with power being dependent on T^4.

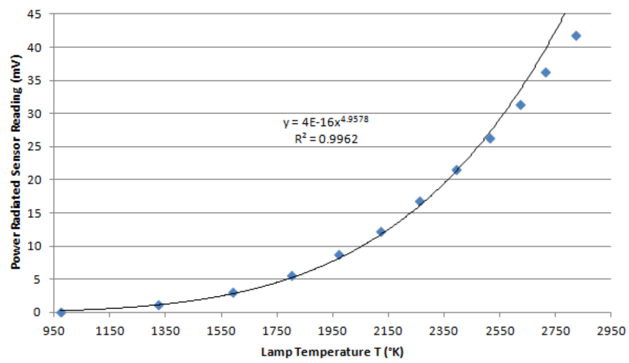

FIG. 4: Thermopile sensor measurements (mV) versus lamp temperature T (°K) with a power regression line with a power of 4.9578

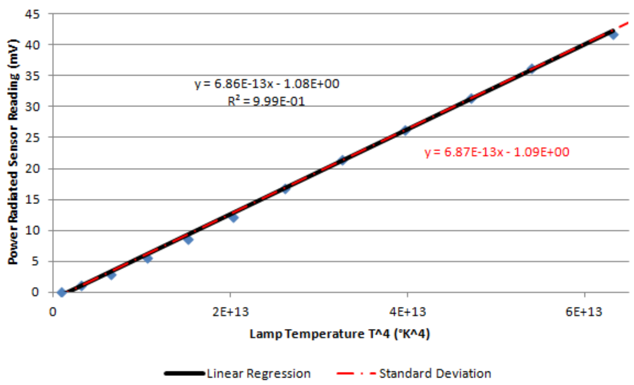

FIG. 5: Thermopile sensor measurements versus lamp temperature T^4 ($^{\circ}K^4$) and the standard deviation line in red (dashed for clarity)

Even more significantly, when power is graphed versus T^4 as seen in Fig.5, the linear relationship confirms the Stefan-Boltzmann Law at high temperatures.

Some sources of error in the calculation of R_T is some resolution in the measurement equipment, trying to measuring a small resistance of the Stefan-Boltzmann lamp at room temperature, having a more accurate device would give more accurate calculations (as it was said, a small error in this measurement will lead to large errors in the filament temperature). In this case I would estimate in a higher bound, the error to be about half the resolution or 0.05, which in turn is about 17% error. In terms of equipment resolution this is the only significant error. Also, the glass in the lamp absorbs and reflects some of the infrared radiation from the filament, this effect will be more dominant at lower temperatures where the infrared is most of the radiation due to Wien's Displacement Law, where the peak of the output is in the lower wavelengths at lower temperatures.

As seen in Fig.6, when taking into account the room temperature, there is almost no difference since the temperature of the lamp is much higher than the ambient temperature, the radiative power at that low temperature doesn't account for any noticeable difference. The Stefan-Boltzmann relation still holds in high-temperature scenarios.

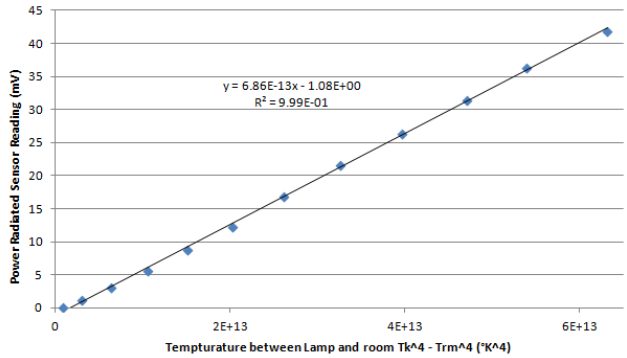

FIG. 6: Thermopile sensor measurements versus temperature difference between the lamp and the room T_k^4 - T_{ref}^4

III. CONCLUSION

This experiment was able to confirm the Stefan-Boltzmann Law, at low temperatures using Leslie Cube's at various temperatures to measuring the infrared radiation of various materials at various temperatures and was confirmed to be dependent on T^4. Similarly by studying these various materials we were able to make calculations and predictions about the emissivity of materials and discovered that materials which absorb radiation better also emit radiation better, and poorer absorbers (better reflectors) make poorer radiators. Using the Stefan-Boltzmann lamp we were also able to confirm the inverse square law and also found that the shape of the lamp filament influences the measurements from the thermopile at closer distances as the filament at that resolution isn't a point source. However, at larger distances the approximation of the lamp as a point source gets better and better leading to confirmation of the inverse square law. The Stefan-Boltzmann Law was also confirmed at high temperatures using the Stefan-Boltzmann lamp and still dependent on T^4. It was found to be in quite good agreement, with some slight deviations due to lower temperature data points and amount of error associated with them. As the temperature of the lamp rises these errors become more irrelevant.

Appendices

Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

TABLE X: Resistance versus Temperature for the Thermal Radiation Cube

R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

TABLE XI: Temperature and Resistivity for Tungsten

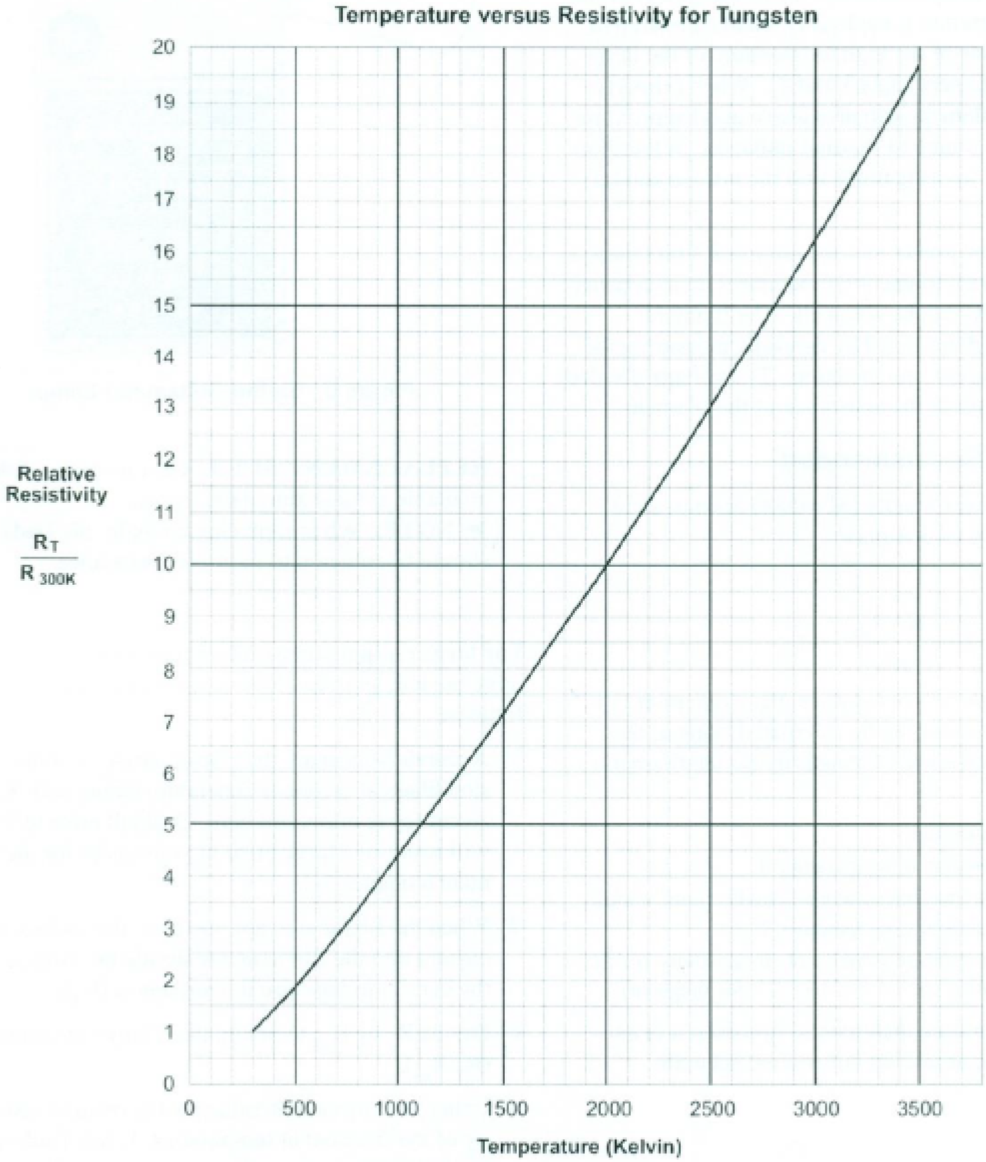

FIG. 7: Temperature and Resistivity for Tungsten

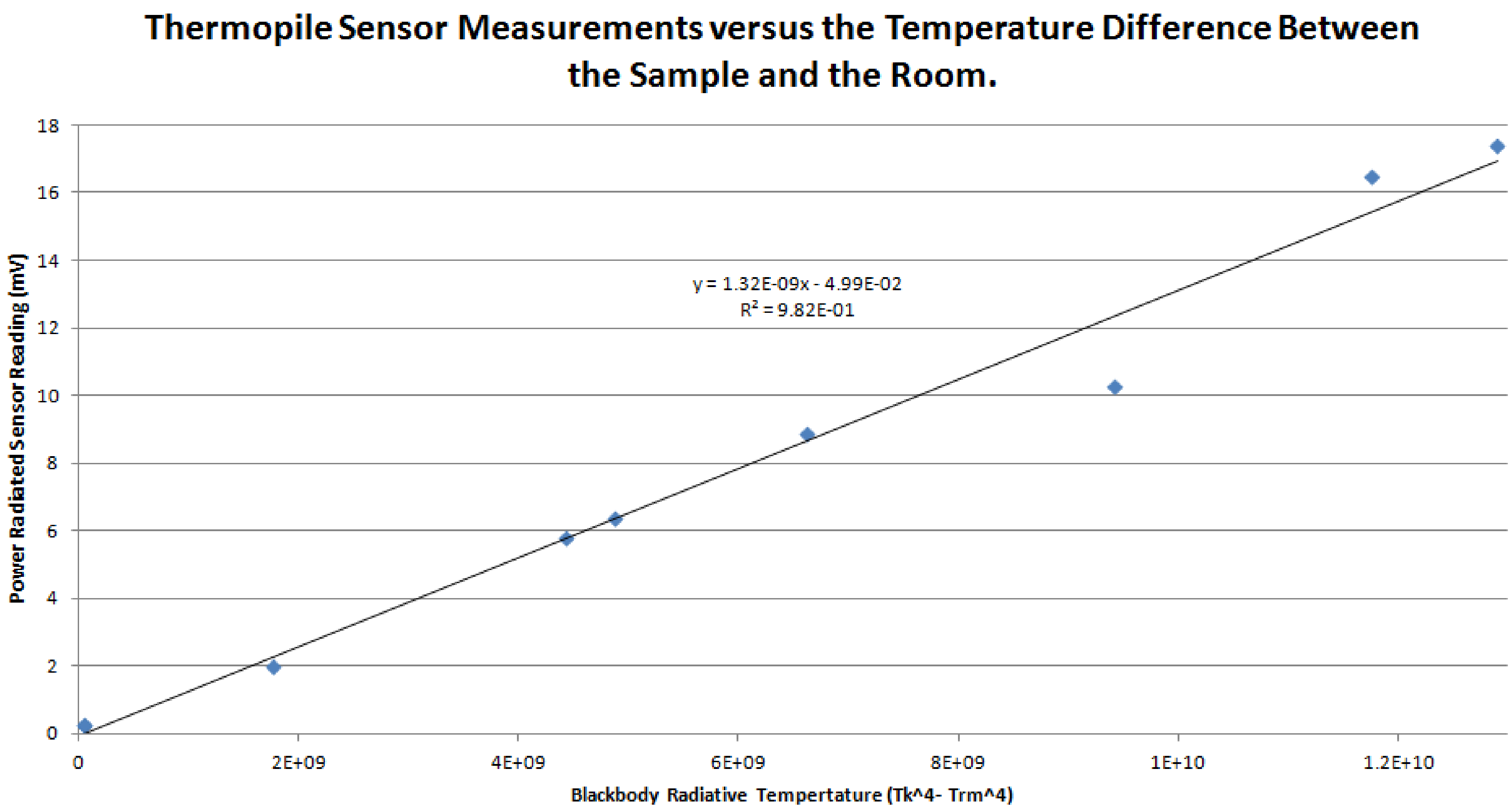

FIG. 1: Thermopile sensor measurements versus the temperature difference between the sample and the room.

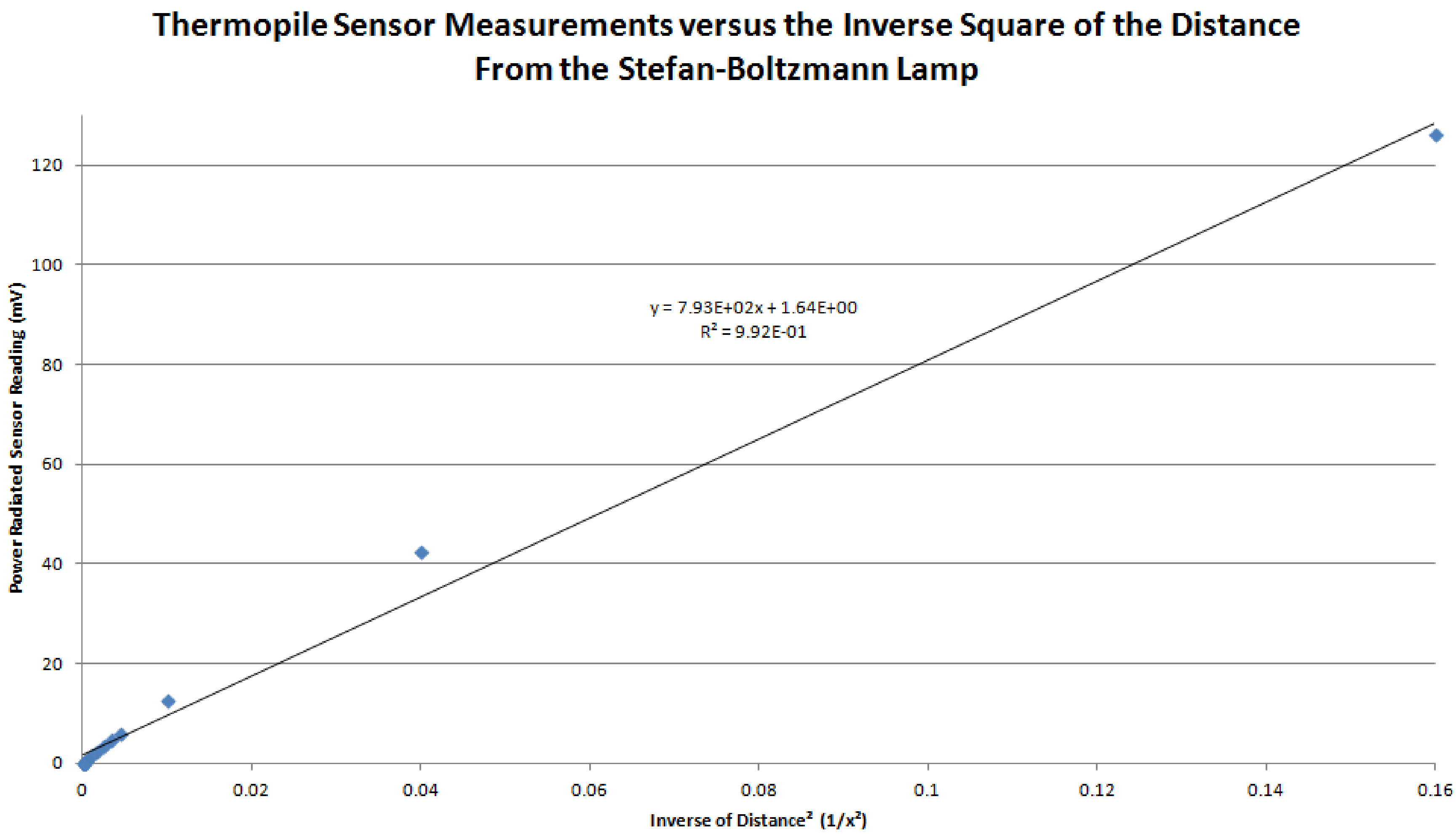

FIG. 2: Thermopile sensor measurements versus the inverse square of the distance from the Stefan-Boltzmann lamp

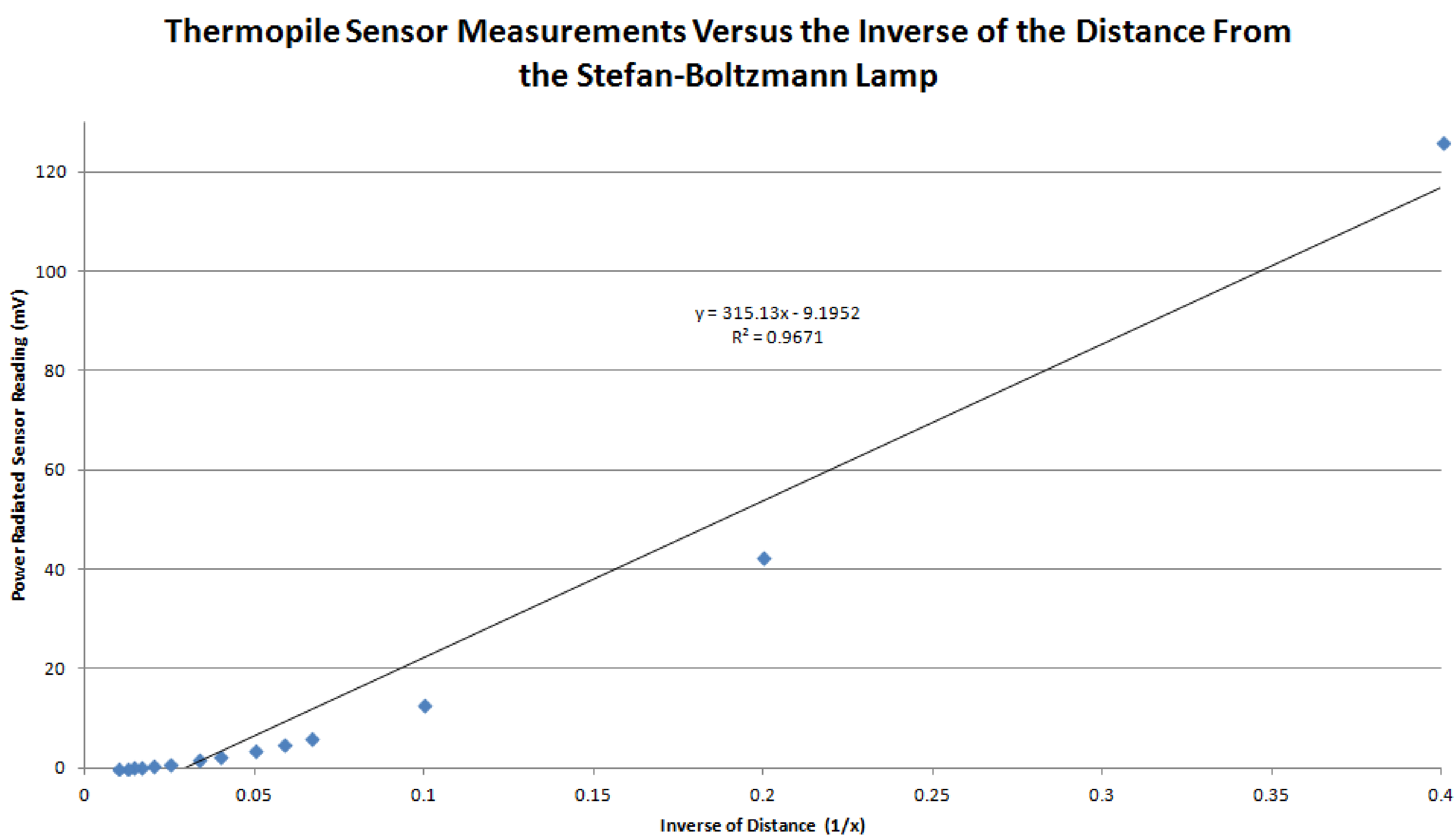

FIG. 3: Thermopile sensor measurements versus the inverse of the distance from the Stefan-Boltzmann lamp

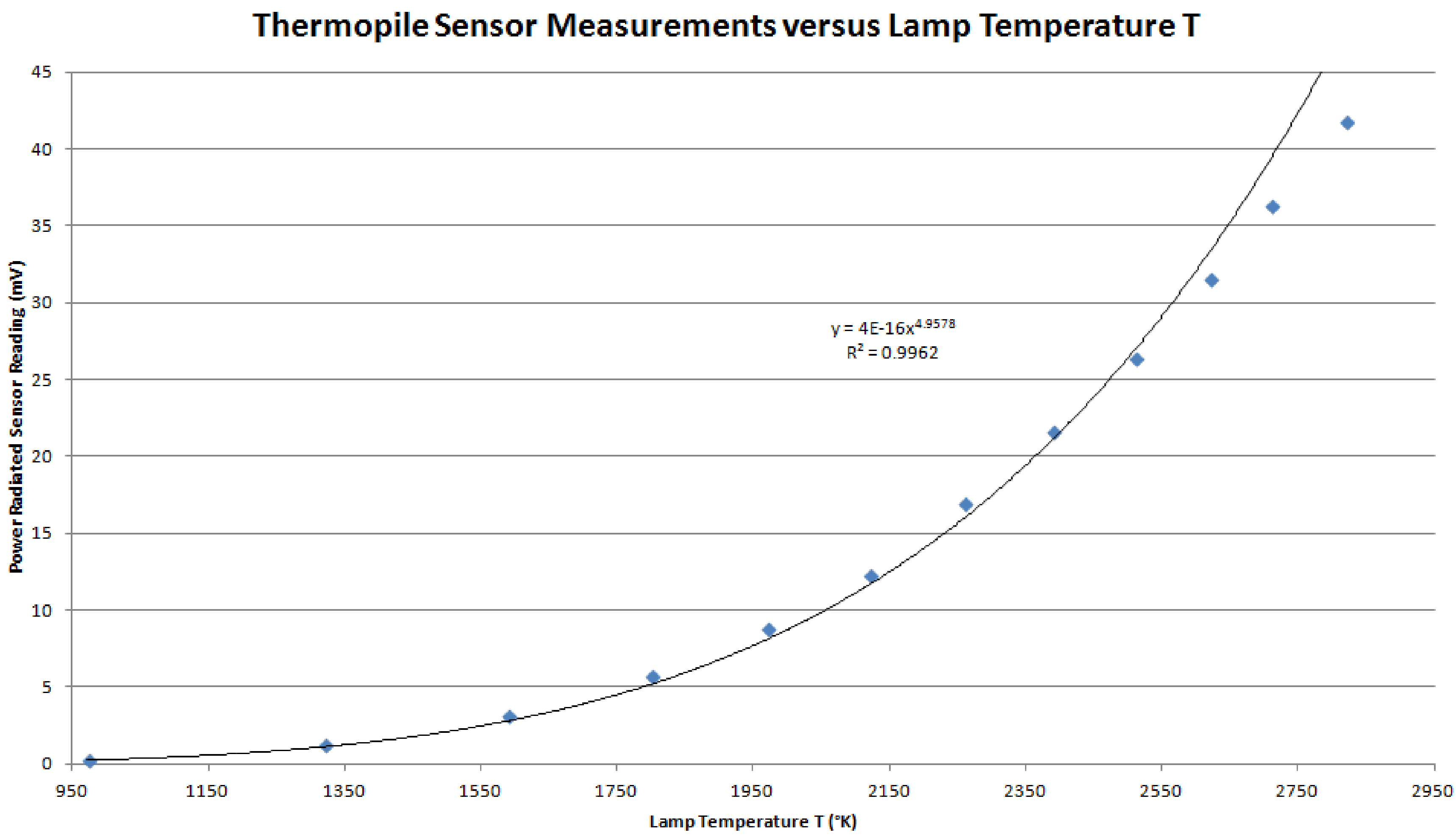

FIG. 4: Thermopile sensor measurements (mV) versus lamp temperature T (°K) with a power regression line with a power of 4.9578

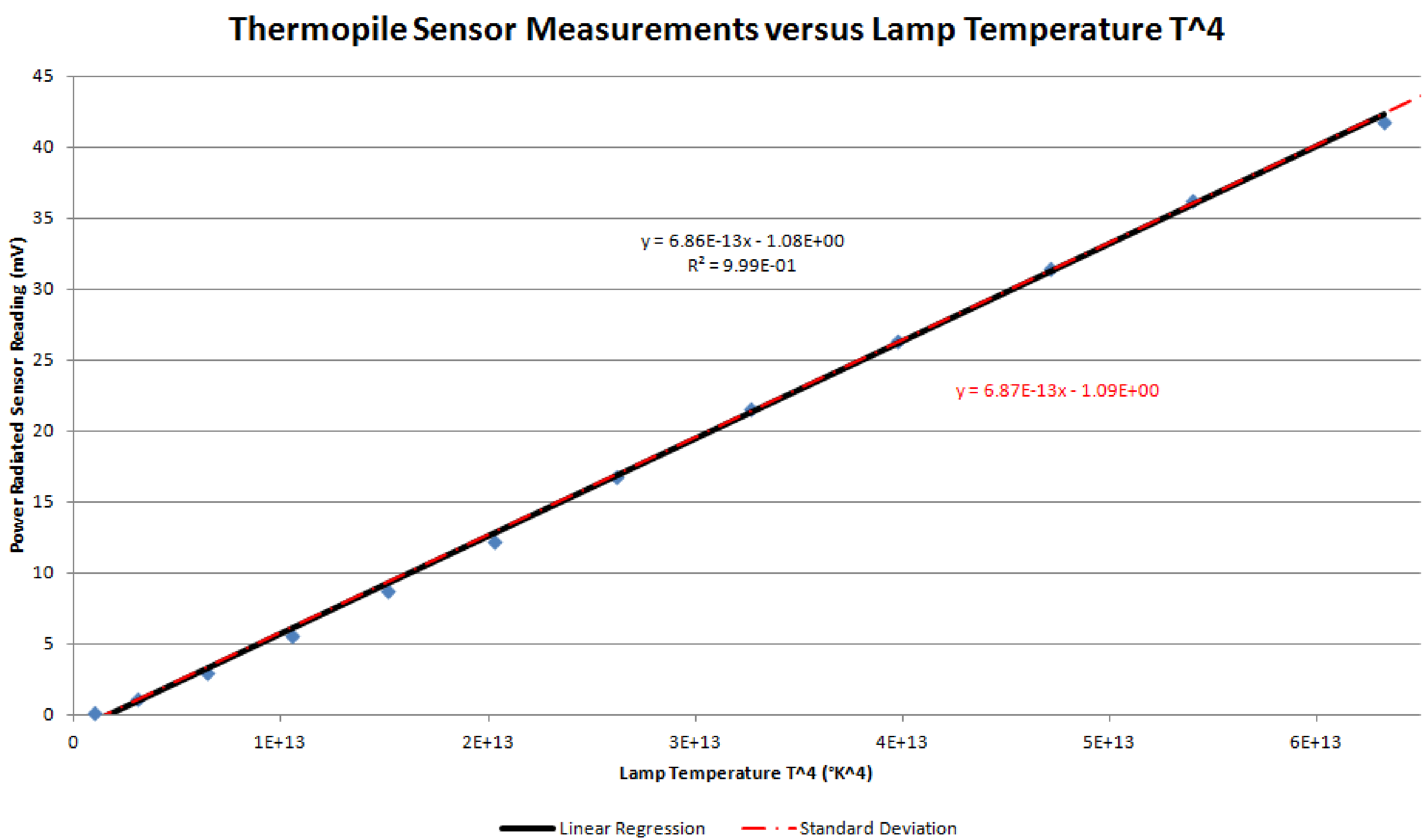

FIG. 5: Thermopile sensor measurements versus lamp temperature T^4 ($^{\circ}K^4$) and the standard deviation line in red (dashed for clarity)

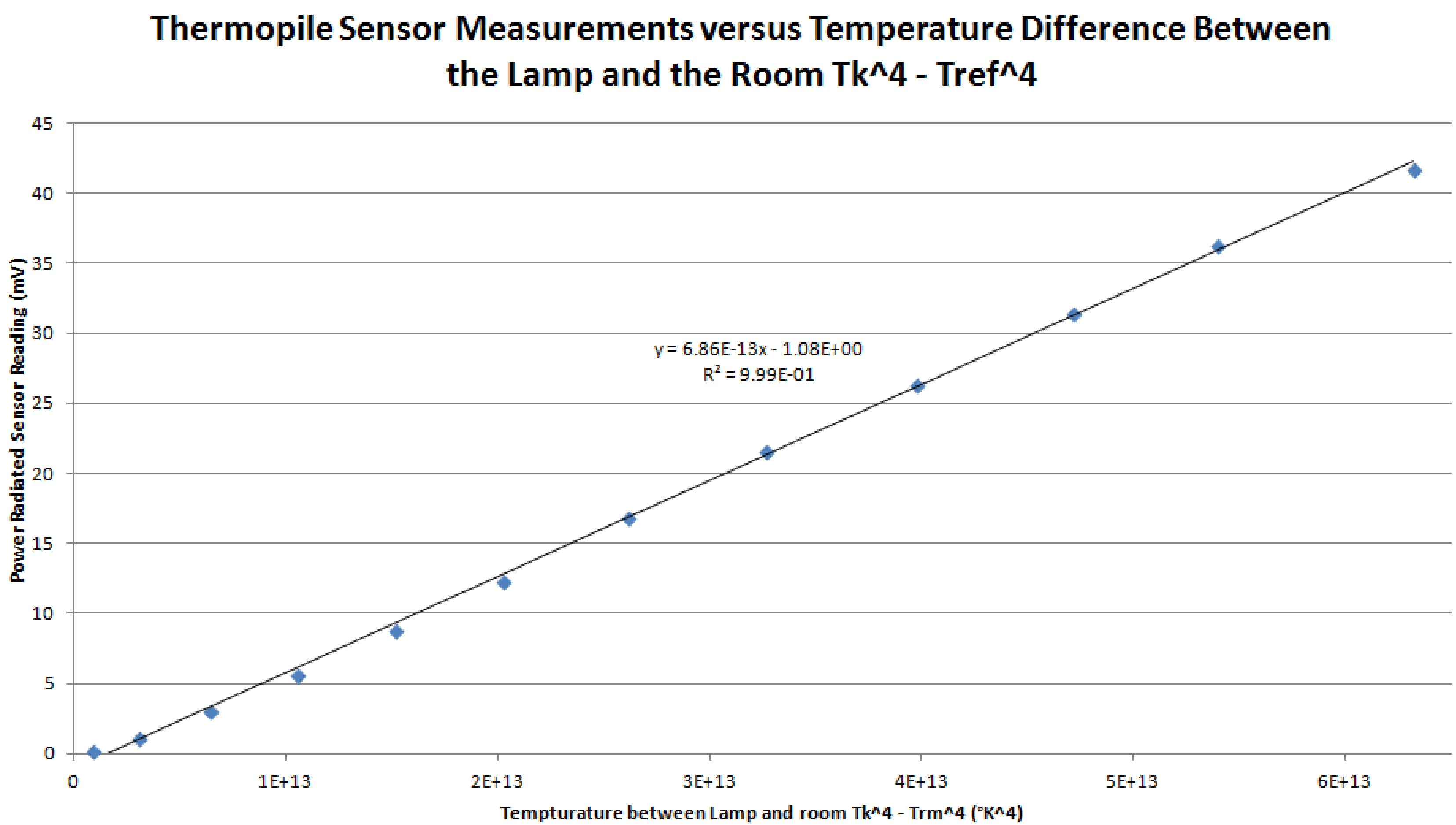

FIG. 6: Thermopile sensor measurements versus temperature difference between the lamp and the room T_k^4 - T_{ref}^4