Electrostatic and Magnetic Deflection of Electrons in a Cathode Ray Tube

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Section 1

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I. PURPOSE

The purpose of this experiment is to observe and verify the properties of electrostatic and magnetic deflection of electrons, such as the relationship of the input voltages, V_A and V_y, to the anode and the deflection plates respectively and the electrostatic deflection D of the electron on the screen of the CRT. Using these properties and known values we will also measure the magnetic deflection of the Earth's field, as well as its direction and magnitude. Finally we will also observe the helical motion of electrons in a magnetic field and the fields ability to focus electrons on the face of the CRT display.

II. ANALYSIS

A. Dependence of D on V_y and V_A

Using a V_A of 443 volts, a focussed spot was on the face of the CRT (cathode ray tube), and the tube was rotated to a position such that the spot was sitting on the x-axis (y=0). The spot moved upwards or downwards as the tube was rotated on the table, and this is due to the Earth's local magnetic field causing the electron to deflect depending on the electrons velocity relative to the field (Eq.1).

$$\vec{F} = e\vec{v} \times \vec{B} \tag{1}$$

Defl. amount (u.g.d.) ±0.125	Defl. plate Voltage V_y (V) $\pm 0.1\% \pm .01V$	$\frac{V_y}{V_A}$
-2.4	-26.65	-0.05857
-2	-21.73	-0.04776
-1	-11.17	-0.02455
1	10.38	0.02281
2	19.69	0.04328
2.6	26.65	0.05857

TABLE I: Vertical deflection of electron beam in CRT using voltage applied to the vertical deflection plates, where 1 u.g.d. (unit graticule divisions) ≡ 0.635 cm, using an accelerating potential V_A of 455 V

Defl. amount (u.g.d.) ±0.125	Defl. plate Voltage V_y (V) $\pm 0.1\% \pm .01V$	$\frac{V_y}{V_A}$
-4	-20.14	-0.09324
-3	-14.15	-0.06551
-2	-9.72	-0.04500
-1	-5.07	-0.02347
1	5.45	0.02523
2	10.64	0.04926
3	15.69	0.07264
4	21.70	0.1004

TABLE II: Vertical deflection of electron beam in CRT using voltage applied to the vertical deflection plates, where 1 u.g.d. (unit graticule divisions) ≡ 0.635 cm, using an accelerating potential V_A of 216 V

Sample Calculations for Table I and II

Using row 1 of Table I:

$$\begin{aligned} V_A &= 455 \\ V_y &= -26.65 \\ \frac{V_y}{V_A} &= \frac{-26.65V}{455V} \\ \frac{V_y}{V_A} &= 0.05857 \end{aligned}$$

The data in Table I and Table II was then graphed (Fig.1), distance deflected D versus input deflection voltage to the vertical deflection plate V_y, such that positive (upward) deflections are associated with positive voltages, and negative(downward) deflections are associated with negative voltages.

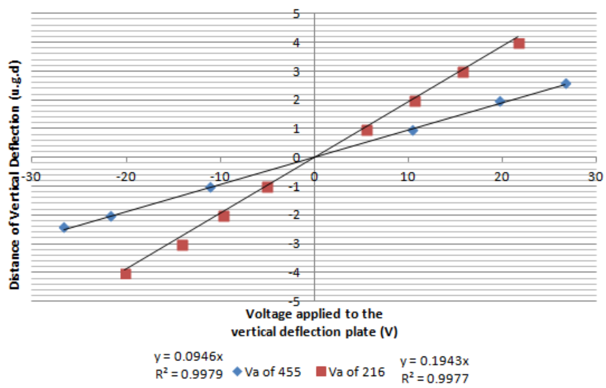

FIG. 1: Distance deflected in u.g.d. versus input deflection voltage to the vertical deflection plate in Volts (D vs. V_y)

These results are expected, as D is linearly proportional to V_y, as seen in Eq.2, which is reflected in the graph with the linear relationship of D versus V_y.

$$D \approx L \tan\theta \approx \frac{L\ell}{2d}\frac{V_y}{V_A} \tag{2}$$

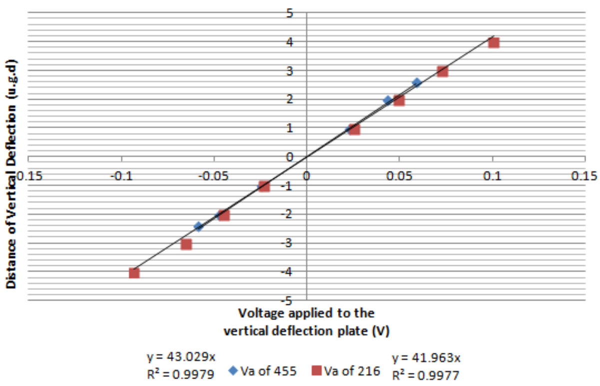

FIG. 2: Deflection versus the ratio of deflection voltage to acceleration voltage (D vs. $\frac{V_y}{V_A}$)

The graphical results of the plot in Fig.2 are as expected, rearranging Eq.2 into Eq.3, you can see that the slope, $\frac{D}{\frac{V_y}{V_A}}$ is constant, since it relies only on the physical properties and dimensions of the CRT: L, ℓ, and d, which are constant.

$$\frac{D}{\frac{V_y}{V_A}} = \frac{L\ell}{2d} \tag{3}$$

B. Creation of an Adjustable Calibrate Voltage Scale

If we consider the DC, deflection coefficient, of the tube where DC is defined as Eq.4, the deflection voltage necessary to deflect the spot 1 u.g.d., we can determine an equation to calculate this property, again rearranging Eq. 3 and substituting DC for $\frac{V_y}{D}$, we get Eq. 5, where DC is essentially some constant, K_{CRT}, dependent on the properties of the CRT, times the anode accelerating potential V_A.

$$DC = \frac{\Delta V_y}{\Delta D} \tag{4}$$

$$DC = \frac{2d}{L\ell}V_A \tag{5}$$

Using the results from Fig.2, and rearranging Eq.2 into Eq. 6, we can see that the inverse of the slope slope of the graph is equal to this constant. Averaging the results from Fig.2 we get an K_{CRT} of $\frac{1}{42.50}$ u.g.d.$^{-1}$.

$$\frac{V_y}{D} = \frac{2d}{L\ell}V_A = K_{CRT}V_A = DC \tag{6}$$

Sample Calculations averaging $\frac{1}{K_{CRT}}$

Using the linear regression data from Fig.2:

$$\begin{aligned} Ave_{slope} &= \frac{Slope_{455}+Slope_{216}}{2} \\ Ave_{slope} &= \frac{43.029+41.963}{2} \\ Ave_{slope} &= \frac{84.992}{2} \\ Ave_{slope} &= 42.496 u.g.d. = \frac{1}{K_{CRT}} \end{aligned}$$

Therefore with a V_A of 455 and 216 as we had for our data in Tables I and II, we get DC's of 10.57 $\pm 0.0111\frac{Volt}{u.g.d.}$ and 5.147 $\pm 0.5.922\times 10^{-3}\frac{Volt}{u.g.d.}$ respectively.

Sample Calculations for DC for V_A of 455

Method 1 using the linear regression data from Fig.1 for V_A of 455:

$$\begin{aligned} Slope &= \frac{D}{V_y} = 0.0946 \\ \frac{1}{Slope} &= \frac{V_y}{D} = \frac{1}{0.0946} \\ DC &= \frac{V_y}{D} = \frac{1}{0.0946} \\ DC &= 10.57\frac{Volt}{u.g.d.} \end{aligned}$$

Uncertainty:

$$\Delta DC = DC \times \frac{\Delta S}{S}$$
$$\Delta DC = 10.57 \times \frac{0.0946*(1-\sqrt{R^2}}{0.0946} = 10.57 \times \frac{0.0946*(1-\sqrt{0.9979}}{0.0946}$$
$$\Delta DC = 0.0111$$

Method 2 using K_{CRT} of 42.50 and Eq.6 and V_A of 455:

$$\begin{aligned} DC &= K_{CRT}V_A \\ DC &= \frac{1}{42.50}\times 455 \\ DC &= 10.70 \end{aligned}$$

We can see that these two values only differ by 1.2%, so derived expression correlated with the experimental observations.

Sample Calculation for Percent Difference of DC's

$$\begin{aligned} \%_{Difference} &= \frac{a-b}{\frac{a+b}{2}}\times 100 \\ \%_{Difference} &= \frac{10.70-10.57}{\frac{10.70+10.57}{2}}\times 100 \\ \%_{Difference} &= \frac{0.13}{10.635}\times 100 \\ \%_{Difference} &= 1.22\% \end{aligned}$$

So to have a DC of 1 $\frac{volt}{u.g.d.}$, we must have a V_A equal to the inverse of K_{CRT}, which is 42.50 Volts. If we have a resistor divider on the vertical deflection plates, where

R_1 is in series with the plates, and R_2 is in parallel, we where the voltage is then defined by Eq. 7.

$$V_eff = \frac{R_2}{R_1 + R_2} V_{in} \tag{7}$$

Using Eq.4 and 7 we can derive an equation for the effective Deflection Coefficient, DC_{eff}:

$$DC_{eff} = \frac{\Delta V_{eff}}{\Delta D}$$
$$DC_{eff} = \frac{\frac{R_2}{R_1+R_2}\cancel{V_{in}}}{\frac{\cancel{V_{in}}}{DC}}$$

$$DC_{eff} = \frac{R_2}{R_1 + R_2} DC \tag{8}$$

$$R_1 = R_2\left(\frac{DC}{DC_{eff}} - 1\right) \tag{9}$$

So assuming we had a DC of 1, to achieve an DC_{eff} of 1, R_1 would have to be zero, and any DC_{eff} above 1 isn't possible if DC is also 1, as R_1 would then have to have a negative resistance, which isn't possible. Assuming we set V_A to 400 volts, we would have a DC of 9.41 (calculated using Eq. 6). So if 1 volt was then placed across the R_2, you would receive the a spot displacement of 9.41×10^{-1} from Eq.4. To have a spot displacement of 1 u.g.d., you would need to supply 9.41 Volts to the deflection plates.

$$A_V = \frac{V_{out}}{V_{in}} \tag{10}$$

If we use a voltage amplifier between the output voltage across R_2 and the input voltage to the deflection plates and using the new V_A resulting in a DC of 9.41, to have a DC_{eff} of 1, we would been an amplifying ratio A_V of 1, as well as any DC_{eff} blow or equal too 9.41. For DC_{eff} above this range, we can derive an equation to calculate A_V using Eq. 4 and 10:

$$DC_{eff} = \frac{V_{out}}{\Delta D}$$
$$DC_{eff} = \frac{\cancel{V_{in}} A_V}{\frac{\cancel{V_{in}}}{DC}}$$
$$DC_{eff} = DC \cdot A_V$$

$$A_V = \frac{DC_{eff}}{DC} \tag{11}$$

So then to have a DC_{eff} of 10, A_V would be 10.6.

Sample Calculations determination of A_V

Using DC of 9.41, and DC_{eff} of 10:

$$A_V = \frac{DC_{eff}}{DC}$$
$$A_V = \frac{10}{9.41}$$
$$A_V = 1.06$$

C. Creation of an Adjustable Calibrate Time Base

To create an adjustable time base that deflects the spot on the face of the CRT in the horizontal direction, V_x will need to have a range from some negative value, to a positive value, so that the spot is deflected on the whole horizontal range of the CRT, in a constant linear fashion with time.

$$\Delta D = \frac{\Delta V_x}{DC} \rightarrow v_x = \frac{\Delta D}{\Delta t} = \frac{1}{DC} \cdot \frac{\Delta V_x}{\Delta t} = \frac{1}{TC} \tag{12}$$

For v_x to be equal to 1 $\frac{u.g.d.}{second}$, using Eq.12, we can see that $\frac{\Delta V_x}{\Delta t}$ must be equal to DC, which in this case with a V_A of 400 is equal to 9.41 $\frac{Volts}{u.g.d}$. That means that TC, is equal to the inverse of v_x which is 1 $\frac{second}{u.g.d.}$. If two points are 4.6 u.g.d. apart, they are also 4.6 second part.

Sample Calculations for time difference between distances

Using TC of 1:

$$\Delta Time = TC \cdot \Delta x_{u.g.d.}$$
$$\Delta Time = 1 \frac{second}{\cancel{u.g.d.}} \cdot 4.6 \cancel{u.g.d.}$$
$$\Delta Time = 4.6 seconds$$

To achieve a time base, TC, of 2 $\frac{second}{u.g.d.}$, using Eq.12, $\frac{\Delta V_x}{\Delta t}$ must equal $\frac{1}{2}$DC, $4.705 \frac{Volts}{second}$, making the spot speed in the x-direction equal to 0.5 $\frac{u.g.d.}{second}$. And for a TC 0.2 $\frac{second}{u.g.d.}$, $\frac{\Delta V_x}{\Delta t}$ must be $\frac{1}{0.2}$, or 5 times greater than DC which equals $47.05 \frac{Volts}{second}$.

Sample Calculations for time base of 0.2$\frac{seconds}{u.g.d.}$

$$\frac{1}{DC} \cdot \frac{\Delta V_x}{\Delta t} = \frac{1}{TC}$$
$$\frac{\Delta V_x}{\Delta t} = \frac{DC}{TC}$$
$$\frac{\Delta V_x}{\Delta t} = \frac{DC}{0.2}$$
$$\frac{\Delta V_x}{\Delta t} = 5 \times DC = 5 \times 9.41 = 47.05$$

D. Motion of Electrons in Magnetic Fields

The CRT was oriented in such a position that there was no spot deflection on the face of the CRT. In this orientation, we measured the tube elevation to be 72.5°, and the tube projection able to be 16.0°. The direction of the velocity vector $\vec{v_x}$ in this instance is parallel to $\vec{B}$, we know this from Eq. 13 and 14.

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{13}$$

$$\| \vec{F} \| = q(\| \vec{v} \| \; \| \vec{B} \| \sin\theta) \tag{14}$$

Since there is no force deflecting the spot on the CRT, the angle between $\vec{v_x}$ and $\vec{B}$ must be 0.

To measure the magnitude of this local magnetic field, we found the maximum deflections on both sides of the graticule origins, we measured a maximum of 3.4 u.g.d., and a minimum of -2.25; meaning the distance between these maxima would be 3.4-(-2.25) = 5.65 u.g.d.. Using Eq.15 we can calculate the magnitude of this local magnetic field $\vec{B}$ to be $1.488 \times 10^{-4} \pm 3.329 \times 10^{-6} T$.

$$D = \frac{eB}{m_e v_x} \frac{L^2}{2} \tag{15}$$

$$v_x = \sqrt{\frac{2eV_A}{m_e}} \tag{16}$$

Sample Calculations for $\| \vec{B} \|$ using Eq.15 and 16

Using V_A of 248V, D of 5.65 u.g.d., and L = 16.0 ± 0.2cm

$$v_x = \sqrt{\frac{2eV_A}{m_e}}$$
$$v_x = \sqrt{\frac{2e \times 248}{m_e}}$$
$$v_x = s\sqrt{8.724 \times 10^{13}}$$
$$v_x = 9.340 \times 10^6$$

$$D = \frac{eB}{m_e v_x} \frac{L^2}{2}$$
$$5.65 u.g.d. \cdot \frac{6.35 \times 10^{-3} m}{u.g.d.} = \frac{eB}{m_e \cdot 9.340 \times 10^6} \frac{0.16^2}{2}$$
$$B = \frac{2}{0.16^2} \cdot \frac{m_e \cdot 9.340 \times 10^6}{e} \cdot 5.65 u.g.d. \cdot \frac{6.35 \times 10^{-3} m}{u.g.d.}$$
$$B = 1.488 \times 10^{-4} T$$

Uncertainty in B:

$$\Delta D = 0.125 u.g.d$$
$$\Delta V_A = 248V \times 0.0003 \pm 1V$$
$$\Delta B = B \cdot \sqrt{(\frac{\Delta D}{D})^2 + (\frac{1}{2} \cdot \frac{\Delta V_A}{V_A})^2 + (2 \cdot \frac{\Delta L}{L})^2}$$
$$\Delta B = 1.488 \times 10^{-4} \cdot \sqrt{(\frac{0.125}{5.65})^2 + (\frac{1}{2} \cdot \frac{1.0744}{248})^2 + (2 \cdot \frac{0.2}{16.0})^2}$$
$$\Delta B = 3.329^{-6} T$$

For the case of maximum deflection, the direction of the velocity vector $\vec{v_x}$ in this instance is perpendicular to $\vec{B}$, we know this from Eq. 13 and 14. The maximum force on the electron from the magnetic field happens when they are 90° apart.

To find the horizontal component of the magnetic field, $\vec{B_H}$, we rotated the CRT flat on the table, and found the maximum deflections in the vertical direction to be -0.3 and 0.4 u.g.d.. The spot is constantly deflected to the left side of the CRT face, and as the CRT is rotated horizontally, the spot moves upward and downward on the CRT face, but no more than 1 u.g.d in either direction.

This is because in this case, to explain the constant deflection to the left side of the CRT, the vertical magnetic component $\vec{B_V}$ is always perpendicular to the velocity of the electrons, by Eq.13 and the right hand rule, we can see that this component $\vec{B_V}$ is upward since the electrons are experience a force to the right in the direction of travel (appears to deflect to the left side of the CRT face).

As for the horizontal component, as you rotate the CRT around on the table, this is the only component of the magnetic field that deflects the electrons upward or downward on the face of the CRT, since this component of the magnetic field lies in the same plane of motion of the electrons in the CRT as it is rotated on the table. If the tube is rotated to face left w.r.t. the front of the room, the beam deflects downward, and upward when the tube is facing to the right w.r.t. the front of the room.A visual representation of this can be seen in Fig.3.

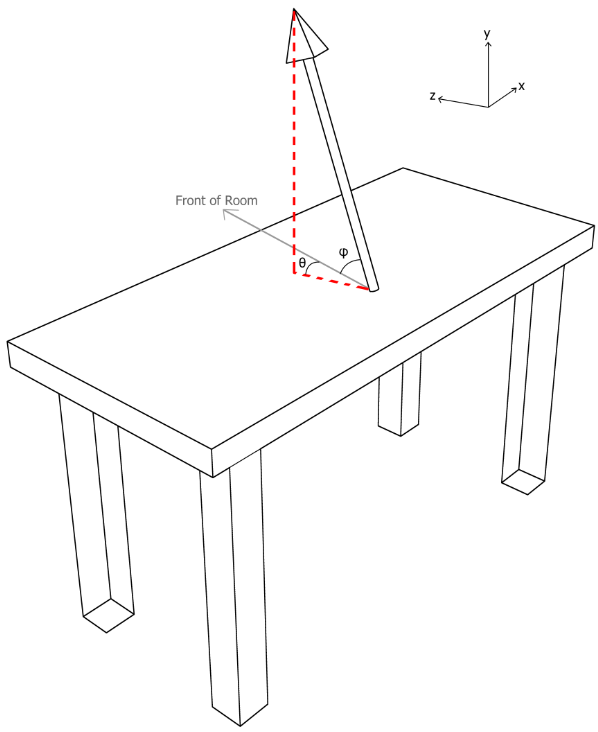

FIG. 3: A visual representation of the local magnetic field, $\vec{B}$, w.r.t. the experiment table and the orientation with the front of the room. Where the measured angles were $\phi = 16.0°$ and $\theta = 72.5°$, and $\vec{B}$ pointing upwards and toward the front of the room.

Using Eq.15, and the distance of 0.4-(-0.3) = 0.7 u.g.d., $\vec{B_H}$ was calculated to being $1.844 \times 10^{-5} \pm 3.325 \times 10^{-6} T$. Comparing the measured value of B_H to the accepted, of $2.1 \times 10^{-5} T$, it was found that there is a percent difference of 12.4%.

Sample Calculations Percent Difference in eperimental vs accepted B_H

Using accepted $B_H of 2.1 \times 10^{-5} T$:

$$\% Difference = \frac{|Accepted - Measured|}{Accepted} \cdot 100$$
$$\% Difference = \frac{|2.1 \times 10^{-5} - 1.84 \times 10^{-5}|}{2.1 \times 10^{-5}} \cdot 100$$

$$\%Difference = 12.4\%$$

Using my measurements of $\vec{B}$ and $\vec{B_H}$, the angle of elevation $\vec{B}$ makes with the plane of the table top is was calculated to be 82.9°. This deviates from the previous measurement of 72.5° by 14.3%.

Sample Calculations for elevation of CRT using $\vec{B}$ and $\vec{B_H}$

$$\cos\theta = \frac{\|\vec{B_H}\|}{\|\vec{B}\|}$$
$$\theta = \arccos\frac{\|\vec{B_H}\|}{\|\vec{B}\|}$$
$$\theta = \arccos\frac{1.844\times10^{-5}}{1.488\times10^{-4}}$$
$$\theta = 82.9^\circ$$

E. Helical Motion of Electrons and Magnetic Focussing

As V_S is increased though its range, while there was a focussed spot on the screen, the spot had a certain path which is travelled along on the CRT face, this shape can be seen in Fig.4. We can see that the electrons initially feel a force upwards as V_{SN} is small, and we can see that their initial velocity is perpendicular to the tube axis in in the -x direction. Again, using Eq.13, and the right hand rule, we can see that the direction of the axial magnetic field is outward, out of the the CRT face.

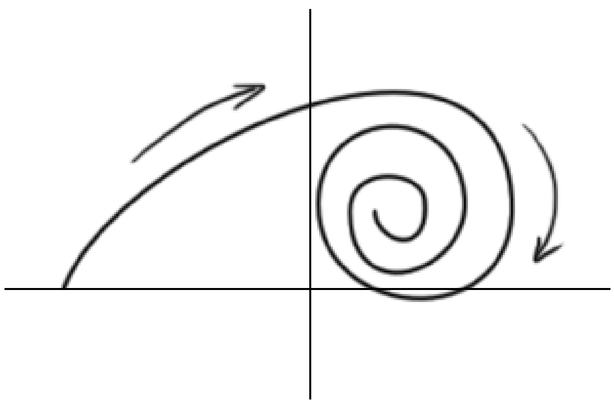

FIG. 4: Sketch of the path of the spot on the face of the CRT, as V_S was increased through it's range to it's maximum value.

Focusing Point N	Voltage V_{SN} (V)
1	13.2
2	24.9
3	36.2
4	47.2
5	56.3
6	65.5

TABLE III: Values of voltages for each corresponding focusing situation.

The values in Table III were graphed, V_{SN} versus N, which showed a linear correlation between the two values. As the relationship between V_{SN} and N is shown in Eq.17, $\frac{V_{SN}}{N}$ is shown to be equal to a constant which is the slope of the graph in Fig.5, which is equal to 11.37 Volts.

$$V_{SN} = (\frac{2\pi v_z}{CL})N \tag{17}$$

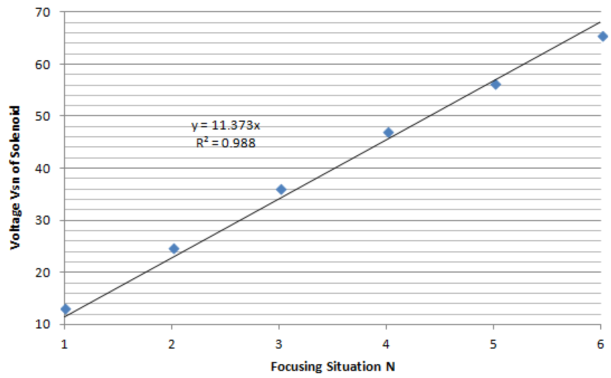

FIG. 5: Graph of V_{SN} versus the focusing situation N

In all, the predicted responses of the beam to the increasing axial magnetic field are realized. As the magnetic field increases, you will have points where the spot is focussed well on the screen, this corresponds with the values in Table III. And in between these points, you can see the beam becomes unfocused, then it will begin to focus again as it reaches the next focussing point, N.

III. DISCUSSION

It was found that the relationship between D and V_Y is constant for a constant V_A, as it was shown in Fig.1 as expected from Eq.2. It was also found that the ratio, $\frac{D}{\frac{V_y}{V_A}}$ was found to be constant, and dependant on the properties and dimensions of the CRT itself, and this value was found graphically in Fig.2, this constant which is equal to $\frac{2d}{L\ell}$ was found to be approximately $\frac{1}{42.50}u.g.d.^{-1}$. Some error in this part of the experiment was the difficulty of reading the values off the face of the CRT for three reasons, as voltage was applied to the deflection plates, the spot became progressively more blurry, due to the graduated face having poor accuracy and lack of finer grid-lines, and being separated from the face of the CRT by some distance as it was more difficult to keep both in focus than if they were closer together, or the screen was printed directly on the glass CRT.

The orientation and magnitude of the Earth's local magnetic field was also found relative to the front of the room and experiment table's surface. We found that the tube had an elevation angle of 72.5° and a projection angle of 16.0°, a graphical representation of this can be seen in Fig.3. The absolute magnitude of $\vec{B}$ was found to be $1.488\times10^{-4}\pm3.329\times10^{-6}T$, we also measured the horizontal component of the local magnetic field, $\vec{B_H}$ was found to be $1.844 \times 10^{-5} \pm 3.325 \times 10^{-6} T$. It was found that there was a difference of 12.4%, but the accepted value of $2.1 \times 10^{-5} T$ was within range of the calculated error. The elevation angle of the tube was calculated again using the measured magnitudes of $\vec{B}$ and $\vec{B_H}$, and was found to be 82.9° which was 14.3% off our previously measured elevation angle of the local magnetic field. Some error encountered in this portion of the experiment again, is the difficulty to read the spots exact position on the graduated face, due to its poor accuracy and trouble to focus on the screen and dot at the same time. But also due to the equipment, it was hard to both position the CRT in some orientation without the use of stabilization equipment made it challenging to get consistent results though this experiment. To counter this problem, some more elaborate equipment can be used, like braces, clamps, or a bracket of some kind.

The helical motion of electrons and the viability of magnetic focusing was also investigated, and it was found to work quite well. The relationship of the Voltage to the solenoid V_S, was found to be linearly correlated with the focusing situations N, this verifies the relationship as seen in Eq.17, and the ratio of $\frac{V_{SN}}{N}$ is defined by a constant. As V_{SN} was increased the spot would focus to a point, then it would become blurry, then focus again, and so on. This also verifies the periodic properties of the helical motion of the electron in the CRT due to the axial magnetic field.

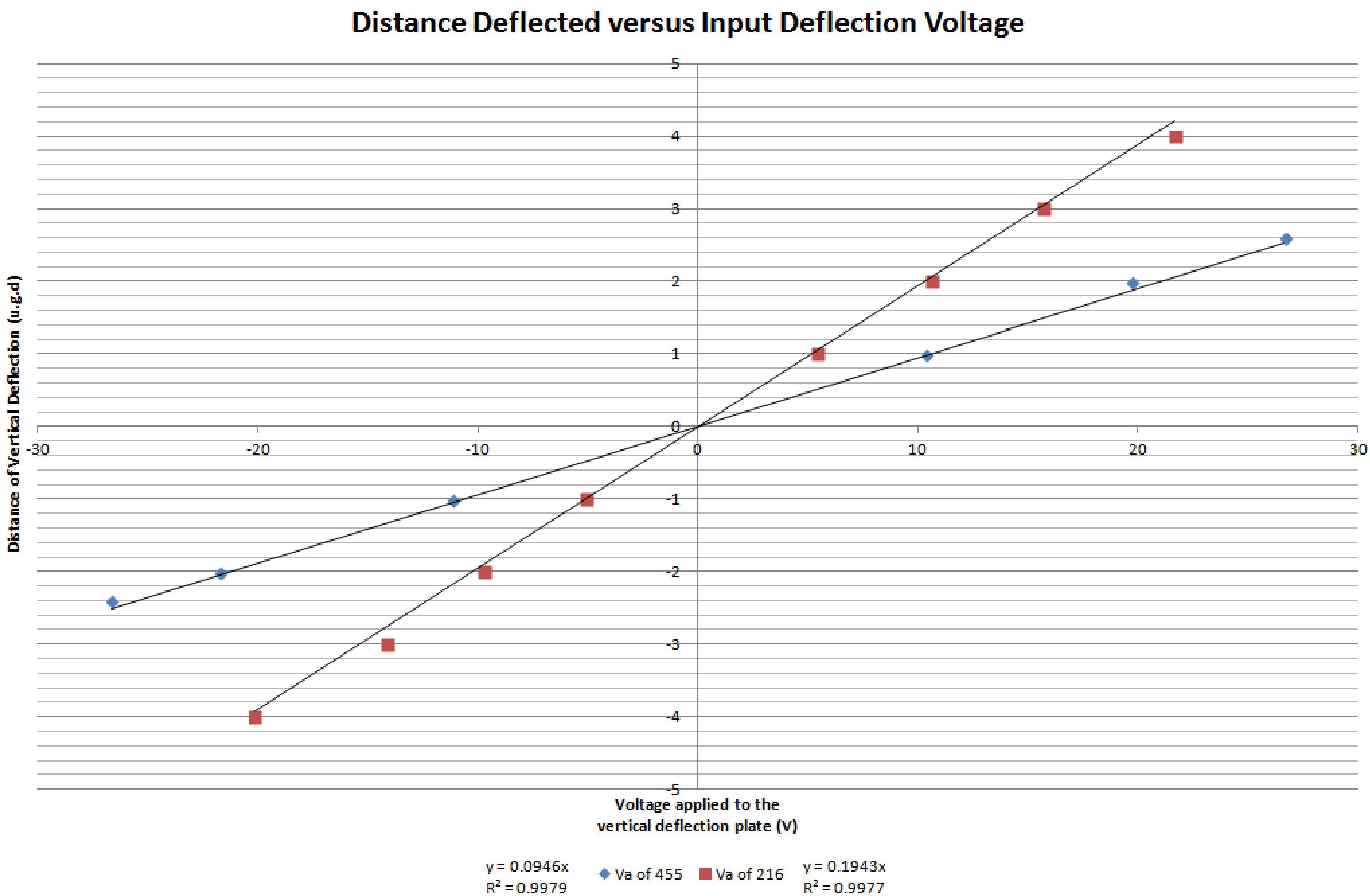

FIG. 1: Distance deflected in u.g.d. versus input deflection voltage to the vertical deflection plate in Volts (D vs. V_y)

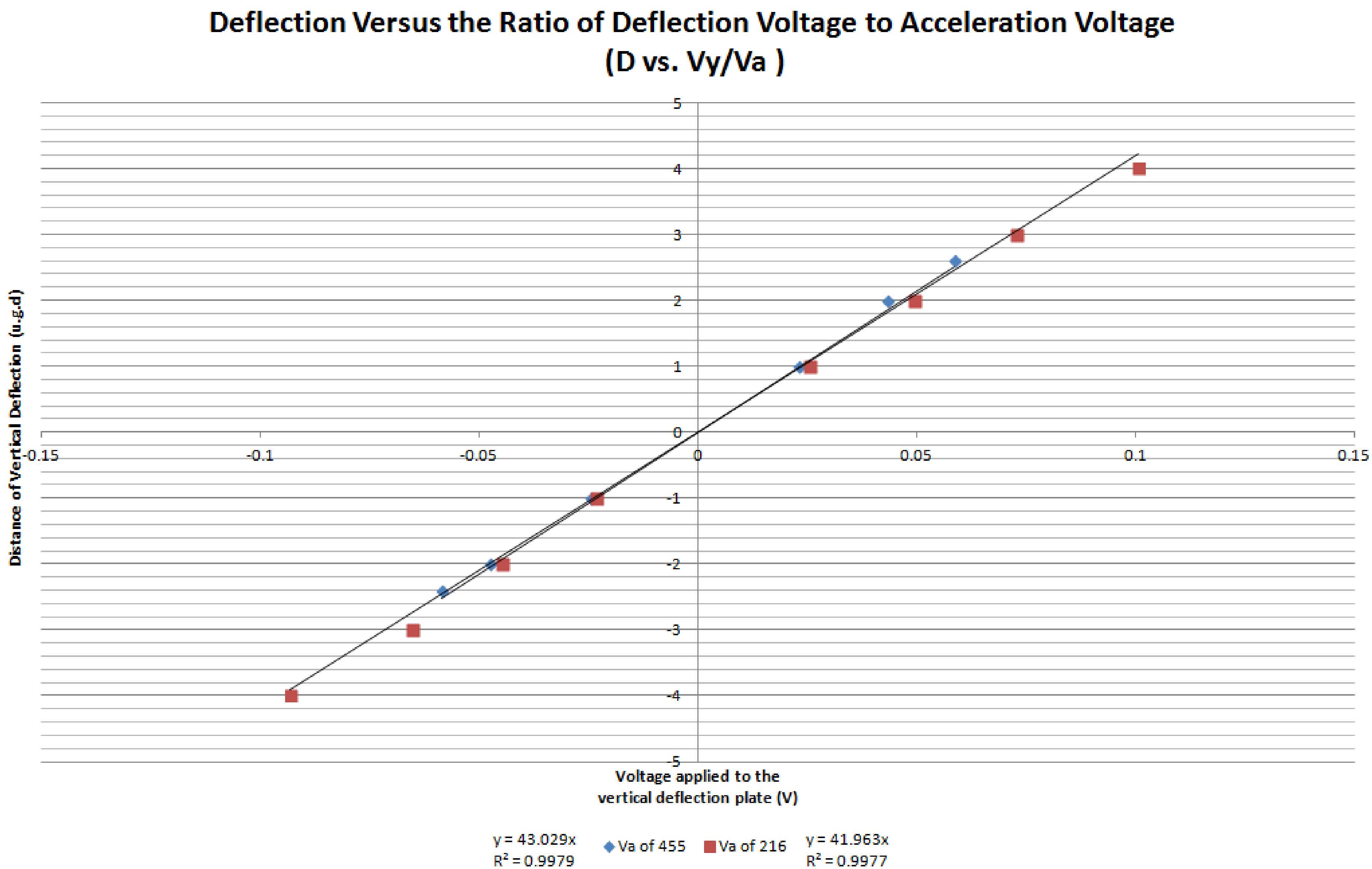

FIG. 2: Deflection versus the ratio of deflection voltage to acceleration voltage (D vs. $\frac{V_y}{V_A}$)

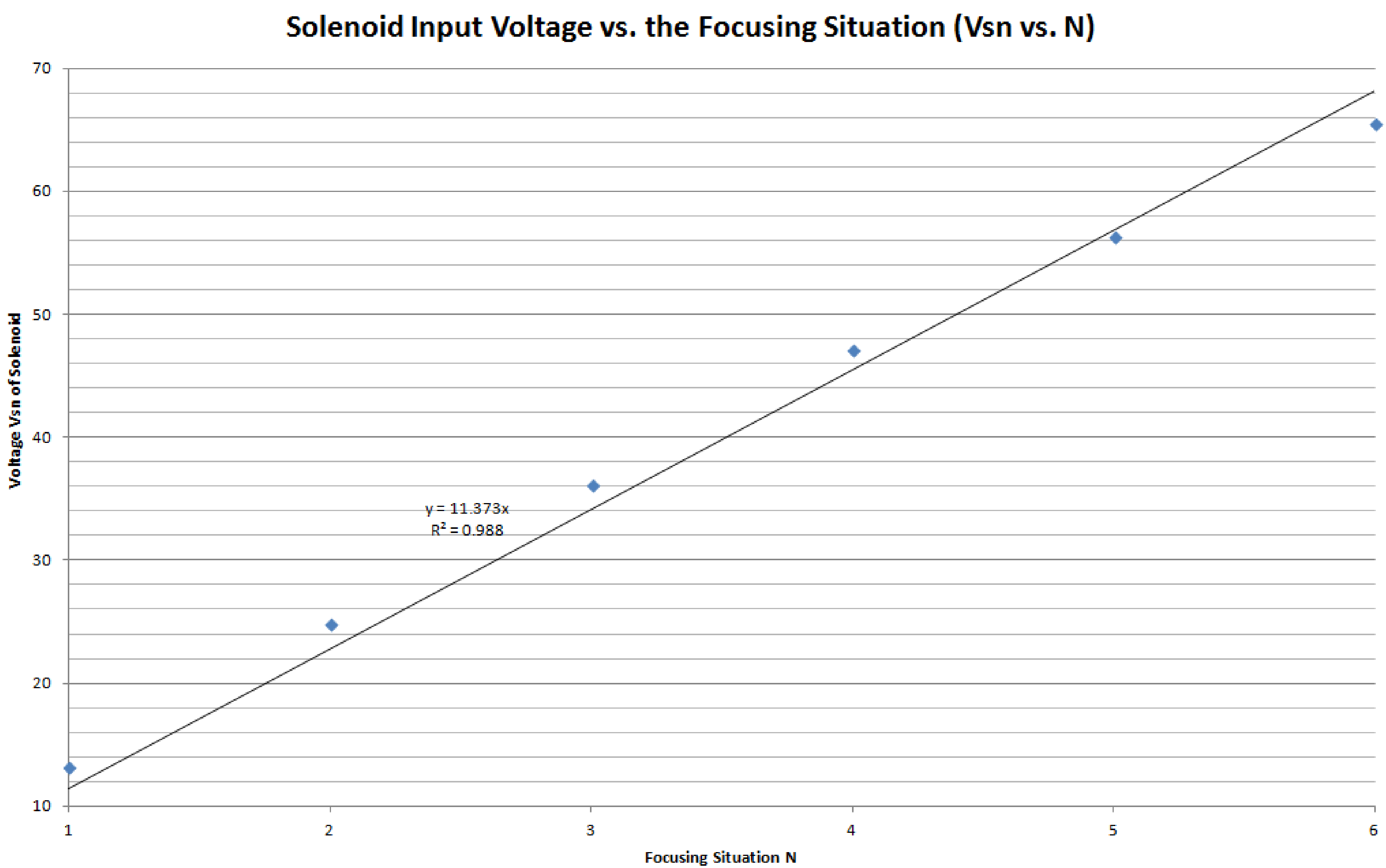

FIG. 5: Graph of V_{SN} versus the focusing situation N