The Measurement of the Charge to Mass Ratio of an Election – Cyclotron

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I. PURPOSE

The purpose of this experiment is to learn how electrons and their trajectories are affected travelling though a Welch no. 623 e/m Vacuum tube under the influence of local magnetic fields and fields produced by in-line Helmholtz coils. The objective of this experiment is to measure the charge to mass ratio for electrons travelling through the vacuum tube using known values and properties between input voltage, the Helmholtz coils, and electromagnetic principles.

II. ANALYSIS

Local Magnetic Field

When the apparatus is properly set up such that the magnetic field produced by the Helmholtz coils are perpendicular to the local magnetic fields to the apparatus, without any current being supplied the electron beam will be deflected toward the south (Fig. 1). Due to the force experienced,

$$\vec{F} = e\vec{v} \times \vec{B} \tag{1}$$

from the local magnetic field (which is mostly due to the Earth's magnetic field). Using Eq. 1 and the observation that the beam is deflected to the south, we can infer that the local Earth's magnetic field is going downward relative to the apparatus.

Filament

When the current on the filament is reversed, the electron beam comes out at an upward angle, or vice-verse depending on the initial set-up of the current supplied to the filament. This is due to the electrons velocity in the wire from the resultant current, if the electrons are travelling up the filament, they have a small initial velocity upward when they are boiled off the filament, or downward if the current is down in the filament.

Coils

As a current is supplied to the Helmholtz coils, if the current in flowing in such a direction such that the induced magnetic field is also downward, this will cause the beam to bend Southward a greater degree proportional to the current flowing through the coils. If the current is reversed such that the induced magnetic field is upwards, the beam will be deflected in the Northward and if the current is great enough, the beam will deflect into a circular shape tending towards the back of the casing that houses the filament which produces the electrons.

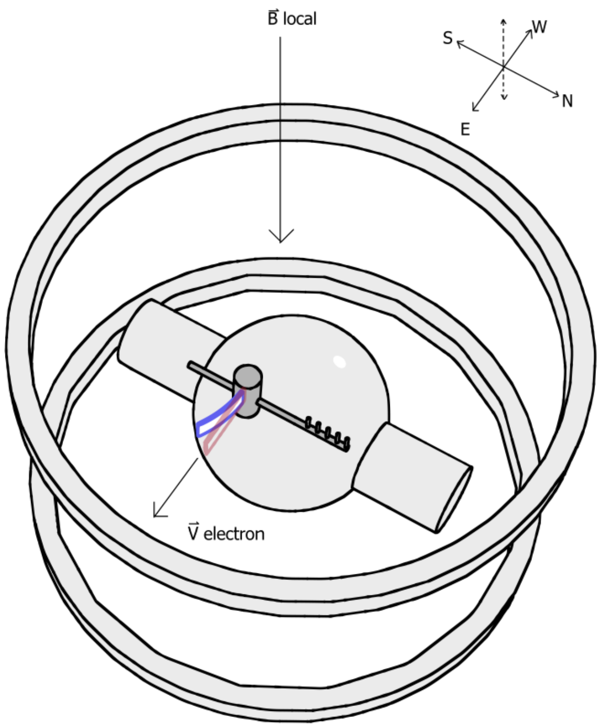

FIG. 1: Diagram showing the apparatus: the two Helmholtz coils with the vacuum tube in the center, with the electron beam coming out towards the East. Also shown is the deflection of the electron beam (blue) Southward and the optimal straight electron beam orientation (red) when in the presence of a local magnetic field of zero should be straight Eastward.

Anode

When the potential voltage to the anode is increased, the beam becomes fuzzy, this is because the electrons are given more energy and as an electron hits a Hg atom in the vacuum tube and ionizes it, the electron still has energy to ionize another Hg atom. As the electrons scatter, they ionize other Hg atoms in the vicinity making the beam appear fuzzy and less sharp. When the anode potential is reduced such that the electrons have enough energy to only ionize one Hg atom, the beam appears much sharper. When the anode potential is significantly reduced, to a point where the electrons being accelerated though the potential difference no longer have sufficient energy to ionize the Hg atoms in the vacuum tube, the beam will disappear. The potential value at which the beam disappears was measured to be 13.05 ± 0.01 Volts.

Calculating the Earth's Magnetic Field

Trials were done, increasing the current supplied to the coils such that they cancelled out the local magnetic field from the Earth affecting the electron beam. The values measured were,

Trial	I (mA ± 0.03% + 1 l.s.d)
1	0.178
2	0.172
3	0.169
4	0.173
5	0.178
6	0.181
7	0.164
8	0.209
9	0.183
10	0.170

TABLE I: Helmholtz currents that cancel out the local magnetic field, causing the electron beam to travel straight

The magnetic field can then be calculated using Eq.2,

$$B = \frac{8\mu_0 NI}{a\sqrt{125}} \tag{2}$$

where a is the radius of the coils (0.33 meters), and N is the number of turns (72). The average magnitude of the local Earth's magnetic field is calculated to be $3.492\times10^{-4} \pm 1.96 \times 10^{-6}$ T.

Calculating the Charge to Mass Ratio

The current needed for the Helmholtz coils to bend the electron beam with different anode potentials were measured,

Anode potential:	20.00 V	30.00 V	40.00 V	50.00 V	55.47 V
Diameter (m)	I (mA)	I (mA)	I (mA)	I (mA)	I (mA)
0.115	1.5914	1.899	2.161	2.383	2.507
0.103	1.753	2.104	2.404	2.640	2.768
0.090	1.978	2.372	2.699	2.986	3.117
0.078	2.267	2.703	3.100	3.422	3.586
0.065	2.666	3.210	3.638	4.024	4.222

TABLE II: Helmholtz currents needed to create a circular electron beam of various diameters, with various anode voltages

The magnetic field strength inside the coil can then be calculated, and the product B^2D^2 can be found, and the average values were calculated to be:

8V versus B^2D^2 can then be graphed to reveal the experimental value of $\frac{e}{m}$, which can be seen in Fig.2, was

Anode V	20.00	30.00	40.00	50.00	55.47
$B^2D^2(\times10^{-9})$	1.068	1.502	1.990	2.455	2.714

TABLE III: B^2D^2 averages for a given anode potential.

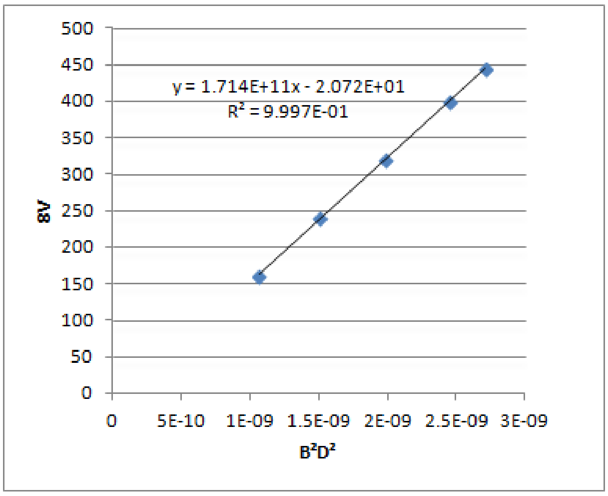

FIG. 2: 8V plotted against B^2D^2. Linear least squares fit with a slope of 1.714×10^{11}.

measured to be 1.714×10^{11} coulomb/kg. The accepted value of $\frac{e}{m}$ is 1.759×10^{11} coulomb/kg.

The error in the current reading is ±(0.03% + 0.001 mA), and the error in the Diameter readings would be about .001 meters, and the precision on the voltage readings is also ±(0.1% + 0.01 V).

% error in B_{earth}
0.0434
0.0441
0.0445
0.0440
0.0434
0.0431
0.452
0.0405
0.0429
0.0444
0.0434

TABLE IV: Error in B_{earth} for each result from TableI using Eq.2

$$\frac{e}{m} = \frac{8V}{B^2D^2} \tag{3}$$

This affects my result by an average of about 0.610%, meaning my final result is $(1.714 \pm 0.01) \times 10^{11}$ coulomb/kg. This doesn't affect my result too significantly.

Anode potential:	20.00 V	30.00 V	40.00 V	50.00 V	55.47 V
	% error in B_D (1×10^{-2})				
	0.0473	0.0422	0.0391	0.0369	0.0359
	0.0444	0.0397	0.0367	0.0348	0.0340
	0.0412	0.0370	0.0344	0.0327	0.0320
	0.0380	0.0344	0.0320	0.0305	0.0299
	0.0347	0.0315	0.0297	0.0283	0.0278

TABLE V: Error in B_D for each result from TableII using Eq.2

Anode potential:	20.00 V	30.00 V	40.00 V	50.00 V	55.47 V
	% error in $B_D^2 D^2$ (1×10^{-2})				
	0.611	0.611	0.610	0.610	0.610

TABLE VI: Error in $B_D^2 D^2$ for each result from TableIV using Eq.3

III. DISCUSSION

The results seem to show a rather convincing result that is quite close to accepted value of $\frac{e}{m}$, with some obvious sources of error, and probably some not so obvious and human sources of error. There is room for improvement with the procedure, quality of the readings, and precision in the measurement tools. However the components of the apparatus are quite ideal for the purpose of this lab, being able to have a uniform magnetic field from the Helmholtz coils and to be able to control it so precisely as as well as knowing what the magnetic field strength is at any point within the two coils allows for an ingenious way to measure $\frac{e}{m}$, using simple E&M interactions with charged particles moving though magnetic fields. This experiment also shows quite well, the relationship between ionizing energy in a very simple manner, by increasing and decreasing the voltage to the anode the electrons kinetic energy and thus ionizing potential and be increased or reduced. The localized B_{earth} was measured to be $3.492\times10^{-4} \pm 1.96\times10^{-6}$ T, which deviated quite a a bit from the expected, around 30%, it may be just that there were interferences. The measured value for $\frac{e}{m}$ was found to be 1.714×10^{11} coulomb/kg which seems quite acceptable deviating only about 3%.