Experiment 1: Measurement of the gyromagnetic ratio

Gyromagnetic ratio of the electron: the ratio of the magnetic dipole moment to the angular momentum of the electron and a fundamental constant of physics.

Classically, a spinning electron has a magnetic moment

$$\mu = I\,A = (-e\,v/\,2\,\pi\,r)\,(\pi\,r^2)$$

and angular momentum $L = m_e\,v\,r$

where r is the radius of the electron, v is electron velocity and A is area.

Then

$$\gamma = (I\,A)\,/\,L = -\,e\,/\,(2m_e)$$

Quantum mechanically, the electron has a spin s = 1/2 and spin angular momentum

$$L_s = (h/\,2\pi)\,(s(s+1))^{1/2}$$

The magnetic dipole moment is

$$\mu_s = -\,g\,e\,(s(s+1))^{1/2}\,(\,h\,/2\,\pi\,m_e\,)$$

where g is called the g-factor and comes from relativistic quantum mechanics as a solution of the Dirac equation. For the electron

$$g = 2\,(1 + (\alpha/\,2\pi) + ...) \qquad (\sim \text{twice as large as the classical result})$$

α is the fine structure constant ≈ 1/137 and measures the strength of the em interaction.

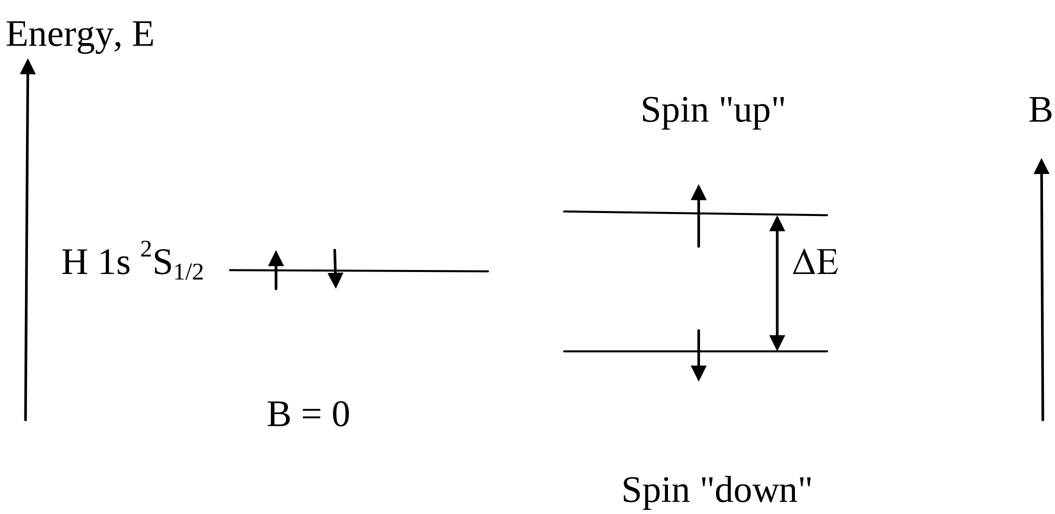
So

$$\gamma_e = (e / m_e) \quad \text{rad/sec tesla}$$

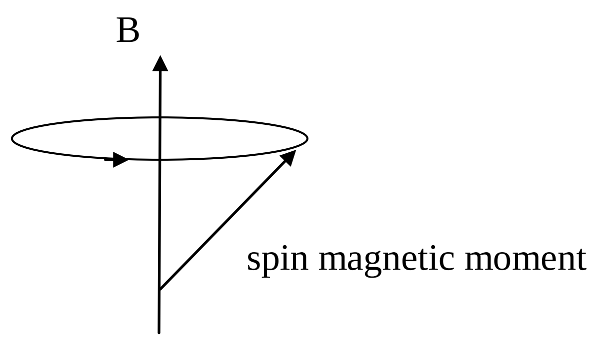
$$\gamma_e / 2\pi = 28.025\text{x}10^3 \text{ MHz / tesla}$$

is the angular precession frequency for an electron spin in an applied magnetic field.

The presence of spin in the electron can be seen in the spectrum of the H atom. The spectrum of an H atom placed in an external B field will show a doubling of its energy levels. The energy difference ΔE is incorrectly predicted from the classical theory



The spins are not completely aligned with the applied B field, but precess at an angular frequency, $\gamma_e B$ about the direction of B.



The Bainbridge experiment enables measurement of e / m_e by measuring the path of an electron with a defined velocity in a magnetic field.

The velocity is obtained by accelerating the electron through a given potential difference.

The final measurement is obtained by imaging the electron path in a gas and recording the radius of the circular path of the electron in an applied field.

Electrons are boiled from a heated filament and accelerated through a potential difference V. then

$$eV = m_e v^2 / 2$$

and

$$e / m_e = v^2 / 2V$$

Typically, V = 20 volts → v = 2.65×10^6 m/sec

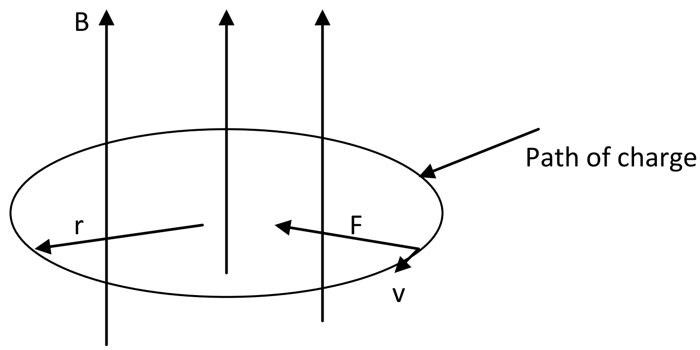
The velocity v can be measured by obtaining the radius r of the electron track in a constant magnetic field, B

The Lorentz force on a charge, q, moving with velocity, **v**, in a magnetic field, **B**, is

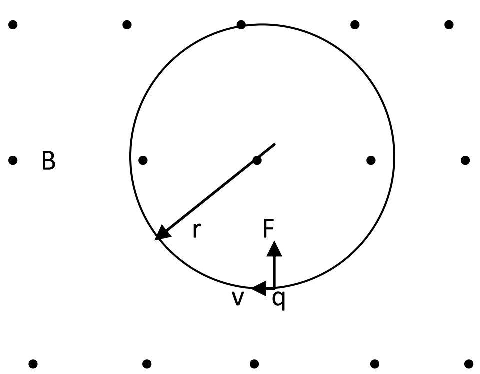
$$\mathbf{F} = q\, \mathbf{v} \times \mathbf{B}$$

Since this force must always be perpendicular to **v**,

a) no work is done by the field B on the charge
b) the charge moves in a circular path in a plane perpendicular to **B**



Looking down a field line it is apparent that the force, **F**, provides a centripetal acceleration as q moves in a circular path around a point in space.



Then (taking the vector cross product and $m = m_e$)

$$(m v^2 / r) = e v B$$

so that $$e / m = (v / B r)$$

The circular path (this only happens in a constant B field) is characterized by the **cyclotron radius**

$$r = (mv) / (qB)$$

and the **cyclotron frequency**,

$$\omega = (v / r) = (q B) / m$$

The **period** of the circular motion, T, is

$$T = (2\pi / \omega) = (2 \pi m) / (q B)$$

For example,

Proton, $v = 10^6$ m/sec with B = 0.1 tesla

$$r = mv / qB = (1.67 \times 10^{-27} \times 10^6) / (1.6 \times 10^{-19} \times 0.1) = 0.104 \text{ meter}$$

$$\omega = v / r = 10^7 \text{ rad/sec}$$

$$T = 2\pi / \omega = 6.3 \times 10^{-7} \text{ sec}$$

Electron, $v = 10^6$ m/sec with B = 0.1 tesla

$r = (9.1x10^{-31} \times 10^{6}) / (1.6 \times 10^{-19} \times 0.1) = 5.9 \times 10^{-5}$ meter

$\omega = 1.8 \times 10^{10}$ rad/sec

$T = 3.6 \times 10^{-10}$ sec

Our value for V is typically 20-60 volts in Expt. 1 and r ≈ 0.1 m, so a much smaller B field is necessary (~ 10^{-4} tesla or 10^{5} nT). This can be obtained from the Helmholtz coils, but compensation is required for the Earth's B field as this is ~ $5x10^{4}$ nT.