The Plane Diffraction Grating

Andy Chmilenko, 20310799
Instructor: Jeff Gardiner
Section 1
(Dated: 2:30 pm Tuesday November 19, 2013)

I. PURPOSE

The purpose of this lab is to observe the diffraction patterns from plane diffraction gratings of two types, transmission gratings and reflection gratings, and confirm the relation between the slit widths, spacing, and the diffraction pattern observed. Using these observations and theory we will also use the observed diffraction pattern from the gratings and known spectral lines to determine the slit separation for the gratings, then the wavelengths of spectral lines from a known source.

II. ANALYSIS

A. The Transmission Grating

Order	Spectral Line	θ_L ($\pm$ 1′ deg)	θ_R ($\pm$ 1′ deg)	θ (rad) ($\pm$ 4.11 $\times$ 10^{-4} rad)	λ (nm)	% Deviation
1^{st}	Hg Blue	119°38′	150°12′	0.2667	435.9 $\pm$ 0.8	0.01%
	Hg Green	115°39′	154°10′	0.3361	545.5 $\pm$ 0.9	1%
	Hg Yellow I	114°30′	155°21′	0.3565	577.2 $\pm$ 0.9	0.04%
	Hg Yellow II	114°26′	155°24′	0.3575	578.8 $\pm$ 0.9	0.05%
	He Red	111°04′	158°40′	0.4154	667.5 $\pm$ 1	0.05%
2^{nd}	Hg Blue	103°05′	166°39′	0.5547	435.6 $\pm$ 0.6	0.05%
	Hg Green	93°31′	176°10′	0.7213	546.1 $\pm$ 0.7	0.005%
	Hg Yellow I 90	90°35′	179°04′	0.7722	577.0 $\pm$ 0.7	0.007%
	Hg Yellow II	90°22′	179°15′	0.7757	579.1 $\pm$ 0.7	0.006%
	He Red	–	–	–	–	–

TABLE I: Left and right angle measurements of the first and second order diffraction patterns for Mercury Blue, Green, Yellow I&II, and Helium Red spectral lines with calculated wavelengths. The second order He Red spectral line could not be observed, it was too dim to make any measurements.

Sample Calculations for θ using row 1 of Table I

$$\theta = \frac{|\theta_L - \theta_R|}{2}$$
$$\theta = \frac{|119°38' - 150°12'|}{2}$$
$$\theta = \frac{|2.08800 - 2.62148|}{2}$$
$$\theta = 0.2667rad$$

Sample Calculations for $\Delta\theta$

$$\Delta\theta = \sqrt{\Delta\theta_L^2 + \Delta\theta_R^2}$$
$$\Delta\theta = \sqrt{(1')^2 + (1')^2}$$
$$\Delta\theta = \pm 4.11 \times 10^{-4} rad$$

Sample Calculations for transmission grating d_{YI} using Row 3 of Table I using Yellow I λ 5770Å

$$\lambda = \frac{d}{m} \cdot \sin\theta$$
$$d = \frac{\lambda \cdot m}{\sin\theta}$$
$$d_{YI} = \frac{5770 \times 10^{-10} \cdot 1}{\sin 0.3565}$$
$$d_{YI} = 1.653 \times 10^{-6} \frac{m}{line}$$

Sample Calculations for Δd_{YI} using Row 3 of Table I

$$\Delta d_{YI} = d_{YI} \cdot \frac{\Delta \sin\theta}{\sin\theta}$$
$$\Delta d_{YI} = d_{YI} \cdot \frac{\cos\theta \Delta\theta}{\sin\theta}$$
$$\Delta d_{YI} = 1.653 \times 10^{-6} \cdot \frac{\cos 0.3565 \cdot 4.11 \times 10^{-4}}{\sin 0.3565}$$
$$\Delta d_{YI} = \pm 1.82 \times 10^{-9} \frac{m}{line}$$

Sample Calculations for transmission grating d_{YII} using Row 4 of Table I using Yellow II λ 5790Å

$$\lambda = \frac{d}{m} \cdot \sin\theta$$
$$d = \frac{\lambda \cdot m}{\sin\theta}$$
$$d_{YII} = \frac{5790 \times 10^{-10} \cdot 1}{\sin 0.3575}$$
$$d_{YII} = 1.655 \times 10^{-6} \frac{m}{line}$$

Sample Calculations for Δd_{YII} using Row 3 of Table I

$$\Delta d_{YII} = d_{YII} \cdot \frac{\Delta \sin\theta}{\sin\theta}$$
$$\Delta d_{YII} = d_{YII} \cdot \frac{\cos\theta \Delta\theta}{\sin\theta}$$
$$\Delta d_{YII} = 1.655 \times 10^{-6} \cdot \frac{\cos 0.3575 \cdot 4.11 \times 10^{-4}}{\sin 0.3575}$$
$$\Delta d_{YII} = \pm 1.82 \times 10^{-9} \frac{m}{line}$$

Sample Calculations for $\overline{d}$

$$\overline{d} = \frac{\sum_i^n d_i}{n}$$
$$\overline{d} = \frac{1.653 + 1.655}{2} \times 10^{-6}$$
$$\overline{d} = 1.654 \times 10^{-6} \frac{m}{line}$$

Sample Calculations for Wavelength λ using Row 1 of Table I using d = 1.654 $\times 10^{-6} \frac{m}{line}$

$$\lambda = \frac{d}{m} \cdot \sin\theta$$
$$\lambda = \frac{1.654 \times 10^{-6}}{1} \cdot \sin 0.2667$$
$$\lambda = 435.9 nm$$

Sample Calculations for $\Delta\lambda$ using Row 1 of Table I

$$\Delta\lambda = \lambda \cdot \sqrt{(\frac{\Delta d}{d})^2 + (\frac{\Delta \sin\theta}{\sin\theta})^2}$$
$$\Delta\lambda = \lambda \cdot \sqrt{(\frac{\Delta d}{d})^2 + (\frac{\cos\theta \Delta\theta}{\sin\theta})^2}$$
$$\Delta\lambda = 435.9 \cdot \sqrt{(\frac{1.82 \times 10^{-9}}{1.654 \times 10^{-6}})^2 + (\frac{\cos 0.2667 \cdot 4.11 \times 10^{-4}}{\sin 0.2667})^2}$$
$$\Delta\lambda = \pm 0.8 nm$$

Sample Calculations for % deviation for the wavelength using Row 1 of Table I, using the accepted value of 4358.35Åfrom Appendix A of the lab manual

$$\%_{deviation} = \frac{|435.9-435.835|}{435.835} \times 100\%$$
$$\%_{deviation} = 0.01\%$$

B. The Reflection Grating

Spectral Line (1^{st} Order)	θ_r ($\pm$ 1′ deg)	θ_N ($\pm$ 1′ deg)	θ (rad) ($\pm$ 4.11 $\times$ 10^{-4} rad)	λ (nm)	% Deviation
Hg Yellow I	249°00′	270°00′	0.3665	577.2 ± 0.9nm	0.04%
Hg Yellow II	249°04′	270°00′	0.3654	578.9 ± 0.9nm	0.03%
He Blue	244°05′	270°00′	0.4523	447.0 ± 0.6nm	0.03%
He Blue-Green	245°49′	270°00′	0.4221	492.2 ± 0.7nm	0.001%
He Yellow	249°22′	270°00′	0.3601	587.1 ± 0.9nm	0.08%
He Red	252°19′	270°00′	0.3086	667.6 ± 1nm	0.03%

TABLE II: Angle measurements of the first order diffraction patterns for Mercury Yellow I&II, and Helium Blue, Blue-Green, Yellow, and Red spectral lines, Angular position of the Normal of the reflection plane, and calculated wavelengths for spectral lines.

Sample Calculations for θ using row 1 of Table II

$$\theta = \theta_N - \theta_r$$
$$\theta = 270°00' - 249°00'$$
$$\theta = 4.7124 - 4.3459$$
$$\theta = 0.3665 rad$$

Sample Calculations for $\Delta\theta$

$$\Delta\theta = \sqrt{\Delta\theta_r^2 + \Delta\theta_N^2}$$
$$\Delta\theta = \sqrt{(1')^2 + (1')^2}$$
$$\Delta\theta = \pm 4.11 \times 10^{-4} rad$$

Sample Calculations for transmission grating d_{YI} using Row 1 of Table II using Yellow I λ 5770Å

$$\lambda = d \cdot (\sin\frac{\pi}{4} - \sin\theta)$$
$$d = \frac{\lambda}{(\sin\frac{\pi}{4} - \sin\theta)}$$
$$d_{YI} = \frac{5770 \times 10^{-10}}{(\sin\frac{\pi}{4} - \sin 0.3665)}$$
$$d_{YI} = 1.654 \times 10^{-6} \frac{m}{line}$$

Sample Calculations for Δd_{YI} using Row 1 of Table II

$$\Delta d_{YI} = d_{YI} \cdot \frac{\Delta \sin\theta}{\sin\theta}$$
$$\Delta d_{YI} = d_{YI} \cdot \frac{\cos\theta\Delta\theta}{\sin\theta}$$
$$\Delta d_{YI} = 1.654 \times 10^{-6} \cdot \frac{\cos 0.3665 \cdot 4.11 \times 10^{-4}}{\sin 0.3665}$$
$$\Delta d_{YI} = \pm 1.77 \times 10^{-9} \frac{m}{line}$$

Sample Calculations for transmission grating d_{YII} using Row 2 of Table II using Yellow II λ 5790Å

$$\lambda = d \cdot (\sin\frac{\pi}{4} - \sin\theta)$$
$$d = \frac{\lambda}{(\sin\frac{\pi}{4} - \sin\theta)}$$
$$d_{YI} = \frac{5790\times10^{-10}}{(\sin\frac{\pi}{4} - \sin 0.3654)}$$
$$d_{YI} = 1.655\times10^{-6}\,\frac{m}{line}$$

Sample Calculations for Δd_{YII} using Row 2 of Table II

$$\Delta d_{YII} = d_{YII}\cdot\frac{\Delta\sin\theta}{\sin\theta}$$
$$\Delta d_{YII} = d_{YII}\cdot\frac{\cos\theta\Delta\theta}{\sin\theta}$$
$$\Delta d_{YII} = 1.655\times10^{-6}\cdot\frac{\cos 0.3654\cdot 4.11\times10^{-4}}{\sin 0.3654}$$
$$\Delta d_{YII} = \pm1.78\times10^{-9}\,\frac{m}{line}$$

Sample Calculations for $\overline{d}$

$$\overline{d} = \frac{\sum_i^n d_i}{n}$$
$$\overline{d} = \frac{1.654+1.655}{2}\times10^{-6}$$
$$\overline{d} = 1.655\times10^{-6}\,\frac{m}{line}$$

Sample Calculations for Wavelength λ using Row 1 of Table II using d = 1.655 $\times10^{-6}\frac{m}{line}$

$$\lambda = d\cdot(\sin\frac{\pi}{4} - \sin\theta)$$
$$\lambda = 1.655\times10^{-6}\cdot(\sin\frac{\pi}{4} - \sin 0.3665)$$
$$\lambda = 577.2nm$$

Sample Calculations for $\Delta\lambda$ using Row 1 of Table II

$$\Delta\lambda = \lambda\cdot\sqrt{(\frac{\Delta d}{d})^2 + (\frac{\Delta\sin\theta}{\sin\theta})^2}$$
$$\Delta\lambda = \lambda\cdot\sqrt{(\frac{\Delta d}{d})^2 + (\frac{\cos\theta\Delta\theta}{\sin\theta})^2}$$
$$\Delta\lambda = 577.2\cdot\sqrt{(\frac{1.78\times10^{-9}}{1.655\times10^{-6}})^2 + (\frac{\cos 0.3665\cdot 4.11\times10^{-4}}{\sin 0.3665})^2}$$
$$\Delta\lambda = \pm0.9nm$$

Sample Calculations for % deviation for the wavelength using Row 3 of Table II, using the accepted value of 4471.48Åfrom Appendix A of the lab manual

$$\%_{deviation} = \frac{|447.0 - 447.148|}{447.148}\times100\%$$
$$\%_{deviation} = 0.03\%$$

III. CONCLUSION

Using a transmission grating, the grating was found to be 1.654 $\times 10^{-6}$ $\pm$ 1.82 $\times 10^{-9}$ $\frac{m}{line}$ ($\approx$6,046 $\frac{lines}{cm}$), and using this value we were able to calculate and and compare the wavelengths for various spectral lines for Hg and He. Our measurements correlated with the accepted values quite well, with an uncertainty of $\pm$0.6-1nm, with values only deviating by 0.006-1%, as outlined in Table I. The second order He Red spectral line could not be observed and was not measured, as it was too dim. Some sources of error are the brightness of the observed spectral lines, especially in the second order spectral lines they became relatively hard to see and measure.

Using a reflection grating, the grating was found to be 1.655 $\times 10^{-6}$ $\pm$ 1.82 $\times 10^{-9}$ $\frac{m}{line}$ ($\approx$ 6,042 $\frac{lines}{cm}$), and using this value we were able to calculate and and compare the wavelengths for various spectral lines for Hg and He. Only first order spectral lines were measured for this part of the experiment, and our measurements correlated with the accepted values quite well again, with uncertainty again ranging from $\pm$0.6-1nm, with values only deviating by 0.001-0.08%, as outlined in Table II.