Fraunhofer Diffraction and Young's Interference

Andy Chmilenko, 20310799
Instructor: Jeff Gardiner
Section 1
(Dated: 2:30 pm Tuesday November 5, 2013)

I. PURPOSE

The purpose of this lab is to observe the diffraction patterns from single and double slit openings and confirm the relation between the slit widths, separation and the diffraction pattern observed. Using these observations and theory we will also use the observed diffraction pattern from a double slit source to estimate the wavelength of the incident light.

II. ANALYSIS

A. The Single Slit

The diffraction pattern was observed as the slit width was adjusted. When the slit becomes larger the diffraction pattern squishes together and the fringes and spaces between become smaller and the pattern becomes slightly sharper. As the slit becomes smaller the pattern stretches out and the fringes and the spaces become thicker and more fuzzy.

Fixed Slit

b_L(± 0.0005cm)	b_r(± 0.0005cm)	b(± 0.0007cm)
3.5512	3.4979	0.0533
3.5411	3.4928	0.0483
3.6222	3.5699	0.0523

TABLE I: Measurements for the left and right bounds respectively of the single slit width using the travelling microscope

Sample Calculations for b using row 1 of Table I

$$b = |b_L - b_R|$$
$$b = |3.5512 - 3.4979|$$
$$b = 0.0553$$

Sample Calculations for Δb

$$\Delta b = \sqrt{\Delta b_L^2 + \Delta b_R^2}$$
$$\Delta b = \sqrt{0.0005^2 + 0.0005cm^2}$$
$$\Delta b = \pm 0.0007cm$$

Sample Calculations for $\bar{b}$

$$\bar{b} = \frac{\sum_i^n b_i}{n}$$
$$\bar{b} = \frac{0.0533+0.0483+0.0523}{3}$$
$$\bar{b} = 0.0513 \times 10^{-2} m$$

Sample Calculations for $\Delta\bar{b}$

$$\Delta\bar{b} = \pm\frac{\Delta b}{\sqrt{n}}$$
$$\Delta\bar{b} = \pm\frac{0.0007}{\sqrt{3}}$$
$$\Delta\bar{b} = \pm 0.0004cm$$

Order of Minimum	d_L ($\pm$ 1′ deg)	d_r ($\pm$ 1′ deg)	d ($\times 10^{-3}$ rad) ($\pm$ 4.11 $\times$ 10^{-4} rad)	Calculated Diffraction Angle($\times 10^{-3}$ rad)	% Deviation
1	189°30′	189°34′	1.16	1.15	0.9%
2	189°26′	189°37′	3.20	2.30	39%
3	189°22′	189°40′	5.24	3.44	52%
4	189°14′	189°43′	8.44	5.74	47%

TABLE II: Measurements for the left and right bounds respectively of orders of diffraction minima for Sodium light and a single slit, as well as calculated quantities such as diffraction angle and deviation from measured diffraction angle.

Sample Calculations for d using row 1 of Table II

$$d = |d_L - d_R|$$
$$d = |189°30' - 189°34'|$$
$$d = |3.30740 - 3.30856|$$
$$d = 1.16 \times 10^{-3} rad$$

Sample Calculations for Δd

$$\Delta d = \sqrt{\Delta d_L^2 + \Delta d_R^2}$$
$$\Delta d = \sqrt{(1')^2 + (1')^2}$$
$$\Delta d = \pm 4.11 \times 10^{-4} rad$$

Sample Calculations for Calculated Diffraction Angle using Row 1 of Table II using λ 5890Å

$$\alpha_n = \frac{n\lambda}{b}$$
$$\alpha_1 = \frac{1 \cdot 5890 \times 10^{-10}}{0.0513 \times 10^{-2}}$$
$$\alpha_1 = 1.148 \times 10^{-3} rad$$

Sample Calculations for % deviation Diffraction Angle using Row 1 of Table II

$$\%_{deviation} = \frac{|1.15 - 1.16|}{1.15} \times 100\%$$
$$\%_{deviation} = 0.9\%$$

White Light Source and Single Slit Diffraction

The white light diffraction pattern has a dark central band, with fringes that had rainbow like patterns on each. This is because each wavelength of light bends at different amounts when going through the single slit.

The Minima are proportional to:

$$\sin\alpha_n = n \cdot \frac{\lambda}{b} \tag{1}$$

So red light, will have a smaller angle of diffraction compared to a larger wavelength colour like blue. This process is similar to refraction in transparent mediums of different indices of refraction.

B. The Double Slit

b_L(± 0.0005cm)	b_r(± 0.0005cm)	b(± 0.0007cm)
Slit 1		
3.1338	3.1467	0.0129
3.1329	3.1249	0.0080
3.1377	3.1463	0.0086
Slit 2		
3.1668	3.1739	0.0071
3.1600	3.1526	0.0074
3.1662	3.1732	0.0070

TABLE III: Measurements for the left and right bounds respectively of the the two slit widths for the double slit using the travelling microscope

Sample Calculations for b using row 1 of Table III

$$b = |b_L - b_R|$$
$$b = |3.1338 - 3.1467|$$
$$b = 0.0129$$

Sample Calculations for Δb

$$\Delta b = \sqrt{\Delta b_L^2 + \Delta b_R^2}$$
$$\Delta b = \sqrt{0.0005^2 + 0.0005cm^2}$$
$$\Delta b = \pm 0.0007cm$$

Sample Calculations for $\bar{b}$

$$\bar{b} = \frac{\sum_i^n b_i}{n}$$
$$\bar{b} = \frac{0.0080+0.0086+0.00710.0074+0.0070}{5}$$
$$\bar{b} = 0.0076 \times 10^{-2}m$$

(Note: Outlier value 0.0129 cm was not used in the calculation for $\bar{b}$)

Sample Calculations for $\Delta\bar{b}$

$$\Delta\bar{b} = \pm\frac{\Delta b}{\sqrt{n}}$$
$$\Delta\bar{b} = \pm\frac{0.0007}{\sqrt{5}}$$
$$\Delta\bar{b} = \pm 0.0003cm$$

Sample Calculations for d to b relation

$$d : b = 0.0076 : 0.0276$$
$$d : b = 1 : 3.6$$

The slit separation to the slit width ratio for the double slit pair is approximately 4.

d_L(± 0.0005cm)	d_r(± 0.0005cm)	d(± 0.0007cm)
Left Edge to Left Edge		
3.4137	3.4418	0.0281
3.4146	3.4415	0.0269
3.4136	3.4424	0.0288
Right Edge to Right Edge		
3.4363	3.4092	0.0271
3.4373	3.4104	0.0269
3.4378	3.4102	0.0276

TABLE IV: Measurements for the left and right bounds respectively of the the two slit separation using the travelling microscope

Sample Calculations for d using row 1 of Table IV

$$d = |d_L - d_R|$$
$$d = |3.4137 - 3.4418|$$
$$d = 0.0281$$

Sample Calculations for Δd

$$\Delta d = \sqrt{\Delta d_L^2 + \Delta d_R^2}$$
$$\Delta d = \sqrt{0.0005^2 + 0.0005cm^2}$$
$$\Delta d = \pm 0.0007cm$$

Sample Calculations for $\overline{d}$

$$\overline{d} = \frac{\sum_i^n d_i}{n}$$
$$\overline{d} = \frac{0.0281+0.0269+0.0288+0.0271+0.0269+0.0276}{6}$$
$$\overline{d} = 0.0276 \times 10^{-2}m$$

Sample Calculations for $\Delta\overline{d}$

$$\Delta\overline{d} = \pm\frac{\Delta d}{\sqrt{n}}$$
$$\Delta\overline{d} = \pm\frac{0.0007}{\sqrt{6}}$$
$$\Delta\overline{d} = \pm 0.0003cm$$

Order of Maximum	M_L (± 1′ deg)	M_r (± 1′ deg)	M ($\times 10^{-2}$ rad) (± 4.11×10^{-4} rad)	Calculated Diffraction Angle($\times 10^{-2}$ rad)	% Deviation
1	190°56′	191°13′	0.247	0.213	16%
2	190°49′	191°19′	0.436	0.427	2%
3	190°44′	191°27′	0.625	0.640	2%
4	–	–	–	–	–
5	190°28′	191°41′	1.062	1.067	0.5%
6	190°20′	191°48′	1.280	1.280	0%
7	190°15′	191°56′	1.469	1.494	1.67%

TABLE V: Measurements for the left and right bounds respectively of orders of diffraction minima for Sodium light and a single slit, as well as calculated quantities such as diffraction angle and deviation from measured diffraction angle.

Sample Calculations for M using row 1 of Table V

$$M = \frac{|M_L - M_R|}{2}$$
$$M = \frac{|190°56' - 191°13'|}{2}$$
$$M = \frac{|3.33242 - 3.33736|}{2}$$
$$M = 0.247 \times 10^{-2} rad$$

Sample Calculations for ΔM

$$\Delta M = \sqrt{\Delta M_L^2 + \Delta M_R^2}$$
$$\Delta M = \sqrt{(1')^2 + (1')^2}$$
$$\Delta M = \pm 4.11 \times 10^{-4} rad$$

Sample Calculations for Calculated Diffraction Angle using Row 1 of Table V using λ 5890Å

$$\alpha_m = \frac{m\lambda}{d}$$
$$\alpha_1 = \frac{1 \cdot 5890 \times 10^{-10}}{0.0276 \times 10^{-2}}$$
$$\alpha_1 = 0.213 \times 10^{-2} rad$$

Sample Calculations for % deviation Diffraction Angle using Row 1 of Table V

$$\%_{deviation} = \frac{|0.213 - 0.247|}{0.213} \times 100\%$$
$$\%_{deviation} = 16\%$$

Laser

Order	Fringe separation (2nt) (cm ± 0.05cm)	Spacing (t) (cm ± 0.05cm)
1	0.95	0.475
2	1.80	0.450
3	2.70	0.450
4	–	–
5	4.60	0.460
6	5.50	0.458
7	6.40	0.457
8	7.30	0.456

TABLE VI: The fringe separation from the left order to the equivalent right order maximum if a He-Ne Laser incident on a double slit with a distance D = 198.5 cm between the slit and the screen.

Sample Calculations for t using row 1 of Table VI

$$2nt = FringeSeparation$$
$$t = \frac{FringeSeparation}{2n}$$
$$t = \frac{0.95}{2 \cdot 1}$$
$$t = 0.475$$

Sample Calculations for $\bar{t}$

$$\bar{t} = \frac{\sum_i^n t_i}{n}$$
$$\bar{t} = \frac{0.475+0.450+0.450+0.460+0.458+0.457+0.456}{7}$$
$$\bar{t} = 0.458 \times 10^{-2} m$$

Sample Calculations for $\Delta\bar{t}$

$$\Delta\bar{t} = \pm\frac{\Delta t}{\sqrt{n}}$$
$$\Delta\bar{t} = \pm\frac{0.05}{\sqrt{7}}$$
$$\Delta\bar{t} = \pm 0.02 cm$$

Sample Calculations for λ using D = 198.5 ± 0.5cm and t = 0.458 ± 0.02cm

$$d \cdot \sin\alpha_m = m\lambda$$
$$\lambda = d \cdot \frac{t}{D}$$
$$\lambda = 0.0276 \times 10^{-2} \cdot \frac{0.458\times10^{-2}}{198.5\times10^{-2}}$$
$$\lambda = 636.8 nm$$

Sample Calculations for $\Delta\lambda$

$$\Delta\lambda = \lambda \cdot \sqrt{(\frac{\delta t}{t})^2 + (\frac{\delta D}{D})^2 + (\frac{\delta d}{d})^2}$$
$$\Delta\lambda = 636.8 \cdot \sqrt{(\frac{0.02}{0.458})^2 + (\frac{0.5}{198.5})^2 + (\frac{0.0003}{0.0276})^2}$$
$$\Delta\lambda = \pm 28.7 nm$$

Sample Calculations for % deviation of λ with the accepted value of 632.8 nm

$$\%_{deviation} = \frac{|632.8-636.8|}{632.8} \times 100\%$$
$$\%_{deviation} = 0.6\%$$

III. CONCLUSION

For single slit diffraction, when the slit width was varied it was observed that the diffraction pattern changed proportions. As the slit was bade larger, the diffraction patter became squished with the fringes and spaces between becoming smaller and the pattern sharper. Conversely as the slit was made smaller the pattern stretches out and the fringes and the spaces become thicker and more fuzzy.

Using a fixed slit, it was found that the slit width, b, to be 0.0513 ± 0.0004 cm, and using this value we were able to calculate and and compare the Diffraction angle of 5890Ålight with our measurements (Table II), and we had mixed results between 0.9% and 52% deviation. The diffraction pattern was fuzzy and rather dim which made it difficult to discern the minima and make accurate measurements.

Using the double slit slide, we were able to measure the slit widths b, and the slit separation d, to be 0.0076 ± 0.0003 cm and 0.0276 ± 0.0003 cm respectively. The ratio of the slit widths to the slit separation was found to be 3.6, which is approximately 4. Using these values we were able to calculate the diffraction angle for specific orders and compare them to our measured angles for these diffraction angles (Table 5). Our results were in rather good agreement only deviating between 0% and 16%. In observations the 4th order maximum for the double slit pattern was not visible, this is due to the single slit envelope having a minimum at the corresponding position where there is a maximum for the double slit.

Using the same double slit pattern, using a laser and measuring the distance between maximum fringes to find the constant maximum separation and knowing the distance from the slits to the screen we were able to calculate the wavelength of the light from the He-Ne laser to be 636.8 ± 28.7 nm. This only differs by 0.6% of the accepted value of 632.8 nm.