The Michelson Interferometer

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Section 1
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I. PURPOSE

The purpose of the experiment is to use the Michelson Interferometer, once calibrated, to measure the wavelength of mercury (Hg) green light, as well as measure the separation of the Hg yellow lines. We will also use the Michelson Interferometer along with a small vacuum chamber to determine the approximate refractive index of air and compare it to our predictions of light travelling through a medium compared to a vacuum. We will also observe the interference pattern of white light and try to apply theory to our observations.

II. ANALYSIS

A. Calibration of the Interferometer in Sodium (Na) Yellow Light

D_1 (± 0.005 mm)	D_2 (± 0.005 mm)	K
5.000	5.160	0.1842 ± 0.0121
6.250	6.400	0.1964 ± 0.0137

TABLE I: Measurements made for the displacement of the micrometer for 100 diffraction lines (Δn) of Na yellow light.

Sample Calculations for K using row 1 of Table I using a λ of 5893Å

$$K = \frac{\Delta n \cdot \lambda}{2 \cdot (|(D_1 - D_2)|}$$
$$K = \frac{100 \cdot 5893A}{2 \cdot (|(5.000 - 5.160)|}$$
$$K = 0.1842$$

Sample Calculations for Δ K using row 1 of Table I

$$\Delta K = K \cdot \sqrt{\frac{(2 \cdot \Delta D)^2}{|D_1 - D_2|^2} + \frac{\Delta(\Delta n)^2}{\Delta n^2}}$$
$$\Delta K = 0.1842\sqrt{\frac{(2 \cdot 0.005)^2}{0.16^2} + \frac{(2)^2}{100^2}}$$
$$\Delta K = \pm 0.0121$$

Sample Calculations for $\bar{K}$ using row 1 of Table I.

$$\bar{K} = \frac{\sum_i^n K_i}{n}$$
$$\bar{K} = \frac{0.1842 + 0.1964}{2}$$
$$\bar{K} = 0.1903$$

Sample Calculations for $\Delta\bar{K}$ using row 1 of Table I.

$$\Delta\bar{K} = \frac{\sum_i^n \Delta K_i}{n}$$
$$\Delta\bar{K} = \frac{0.0121+0.0137}{2}$$
$$\Delta\bar{K} = \pm 0.0129$$

K was found to be 0.1903 ± 0.0129, taking 0.005mm as the uncertainty in the micrometer measurement and 2 as the uncertainty Δn, misreading at most the position of the first counted and last counted diffraction lines.

B. Determination of the Wavelength of the Mercury (Hg) Green Light

D_1 (± 0.005 mm)	D_2 (± 0.005 mm)	λ (nm)
6.300	6.445	551.9 ± 53.4
3.090	3.230	532.8 ± 52.5

TABLE II: Measurements made for the displacement of the micrometer for 100 diffraction lines (Δn) of Hg green light.

Sample Calculations for K using row 1 of Table II using a λ of 5893Å

$$\lambda = \frac{2 \cdot K \cdot (|(D_1 - D_2)|}{\Delta n}$$
$$\lambda = \frac{2 \cdot 0.1903 \cdot (|(6.300 - 6.445)|}{100}$$
$$\lambda = 551.9nm$$

Sample Calculations for $\Delta\lambda$ using row 1 of Table II

$$\Delta\lambda = \lambda \cdot \sqrt{\frac{(2 \cdot \Delta D)^2}{|D_1 - D_2|^2} + \frac{\Delta K^2}{K^2}}$$
$$\Delta\lambda = 551.9\sqrt{\frac{(2 \cdot 0.005)^2}{0.145^2} + \frac{(0.0129)^2}{0.1903^2}}$$
$$\Delta\lambda = \pm 53.4$$

Sample Calculations for $\bar{\lambda}$ using row 1 of Table II.

$$\bar{\lambda} = \frac{\sum_i^n \lambda_i}{n}$$
$$\bar{\lambda} = \frac{551.9+532.8}{2}$$
$$\bar{\lambda} = 542.35$$

Sample Calculations for $\Delta\bar{\lambda}$ using row 1 of Table II.

$$\Delta\bar{\lambda} = \frac{\sum_i^n \Delta\lambda_i}{n}$$
$$\Delta\bar{\lambda} = \frac{53.4+52.5}{2}$$
$$\Delta\bar{\lambda} = \pm 53.0$$

Sample Calculations for % deviation of observed measured wavelength of green light from Hg with the accepted value of 546.074 nm

$$\%_{deviation} = \frac{|546.074 - 542.35|}{546.074} \times 100\%$$
$$\%_{deviation} = 0.7\%$$

λ for the green light emitted by Hg was found to be 542.35 ± 53.0 nm, which only differed by 0.7% from the accepted value of 546.074nm and was well within our calculated uncertainty.

Separation of the Hg Yellow Lines

D_{1-10} (± 0.005 mm): 3.330, 3.740, 4.140, 4.570, 4.980, 5.380, 5.810, 6.220, 6.630, 7.050

δD_{1-9} ($\pm\sqrt{2}\cdot 0.005$ mm): 0.410, 0.400, 0.430, 0.410, 0.400, 0.430, 0.410, 0.410, 0.420

Sample Calculations for $\Delta\overline{\delta D}$

$$\Delta\overline{\delta D} = \frac{\sum_i^n \delta D_i}{n}$$
$$\Delta\overline{\delta D} = \frac{0.410+0.400+0.430+0.410+0.400+0.430+0.410+0.410+0.420}{9}$$
$$\Delta\overline{\delta D} = \pm 0.413$$

Sample Calculations for $\Delta\overline{\delta D}$

$$\Delta\overline{\delta D} = \pm\frac{\Delta\delta D}{\sqrt{n}}$$
$$\Delta\overline{\delta D} = \pm\frac{\sqrt{2}\cdot 0.005}{\sqrt{9}}$$
$$\Delta\overline{\delta D} = \pm 0.00236mm$$

Sample Calculations for $\lambda'' - \lambda'$ using $\overline{\lambda}$ as 578.0 nm

$$\lambda'' - \lambda' = \frac{\overline{\lambda}^2}{2\cdot K\cdot \delta D}$$
$$\lambda'' - \lambda' = \frac{578.0nm^2}{2\cdot 0.1903\cdot 0.413mm}$$
$$\lambda'' - \lambda' = 2.125nm$$

Sample Calculations for $\Delta(\lambda'' - \lambda')$

$$\Delta(\lambda'' - \lambda') = \lambda'' - \lambda' \cdot \sqrt{(\frac{\Delta K}{K})^2(\frac{\Delta\delta D}{\delta D})^2}$$
$$\Delta(\lambda'' - \lambda') = 2.125nm \cdot \sqrt{(\frac{0.0129}{0.1903})^2(\frac{0.00236}{0.413})^2}$$
$$\Delta(\lambda'' - \lambda') = 0.14nm$$

Sample Calculations for % deviation of observed measured wavelength difference of Hg Yellow I and II lines with the accepted difference of 579.065nm-576.959nm = 2.106nm

$$\%_{deviation} = \frac{|2.106 - 2.125|}{2.106} \times 100\%$$
$$\%_{deviation} = 0.9\%$$

$\lambda'' - \lambda'$ for the Hg Yellow I and II lines was measured to be 2.125 ± 0.14nm, which only differed by 0.9% compared to the accepted values of 579.065nm-576.959nm = 2.106nm, the accepted value was also within our estimated uncertainty.

C. The Index of Refraction of Air (for Na Yellow Light)

P_0 (± 0.25 cmHg)	Δn	P_i (± 0.25 cmHg)							
75.0	5	66.0	58	48	39	30	14	–	–
74.0	5	65	56.5	49	41	33	24	15	7
74.0	5	65	60	50	41	31	21	13	–

TABLE III: Measurements taken recording the cell pressure P_i for each interval of 5 (Δn) rings disappearing.

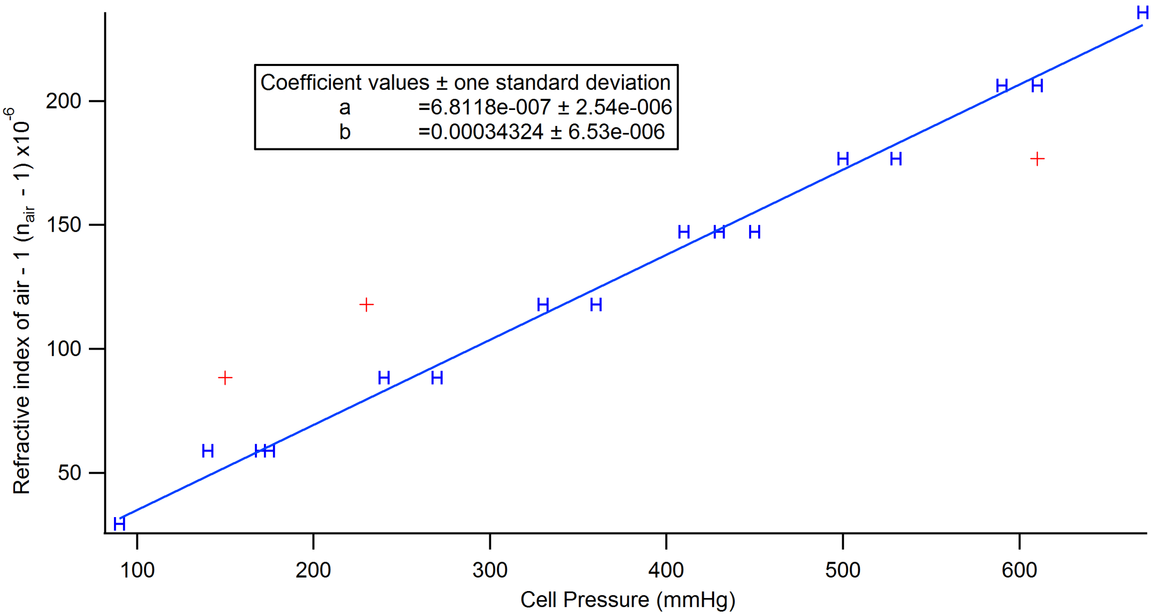

FIG. 1: Refractive index of air - 1 versus the internal cell pressure in mmHg shown in blue. 3 outlier values indicated in red are not used in the regression.

Sample Calculations for n_{air} using regression constants a = 6.812$\times 10^{-7}$ and b = 0.0003432 at standard atmosphere pressure of 760mmHg for Na yellow light

$$n_{air} - 1 = ax + b$$
$$n_{air} - 1 = 6.812 \times 10^{-7} \cdot 760mm + 0.0003432$$
$$n_{air} = 1.00034$$

Sample Calculations for Δn_{air}

$$\Delta n_{air} = (n_{air} - 1) \cdot \sqrt{(\frac{\Delta a}{a})^2 (\frac{\Delta b}{b})^2}$$
$$\Delta n_{air} = 0.00034 \cdot \sqrt{(\frac{2.54}{6.812})^2 (\frac{6.53}{343.2})^2}$$
$$\Delta n_{air} = 0.00013$$

Using the regression constants a and b indicated in Fig.1, we can calculate the refractive index in air for Na yellow light (5893Å) at standard atmospheric pressure (760 mmHg) was found to be 1.00034 ± 0.00013 which is consistent with theory as it should be greater than 1 (refractive index at a vacuum).

D. Observation of White Light Fringes

Trial 1: D_1 = 14.14 ±0.005 mm D_2 = 14.11 ±0.005 mm
Trial 2: D_1 = 14.14 ±0.005 mm D_2 = 14.11 ±0.005 mm

Sample Calculations carriage distance travelled.

$$\delta D_{carriage} = K \cdot \delta D_{micrometer}$$
$$\delta D_{carriage} = 0.1903 \cdot (14.14mm - 14.11mm)$$
$$\delta D_{carriage} = 5.709 \times 10^{-6} m$$

Sample Calculations for $\Delta\delta D_{carriage}$

$$\Delta\delta D_{carriage} = \delta D_{carriage} \cdot \sqrt{\frac{(2\cdot\Delta D)^2}{|D_1 - D_2|^2} + \frac{\Delta K^2}{K^2}}$$
$$\Delta\delta D_{carriage} = 5.709\mu m \cdot \sqrt{\frac{(2\cdot 0.005)^2}{0.03^2} + \frac{0.0129^2}{0.1903^2}}$$
$$\Delta\delta D_{carriage} = 0.34\mu m$$

The white light fringes was observed to be a white band, with a purpler colour, then blue, green, yellow, orange, red, then the pattern began to repeat in the fashion. The approximate distance over which fringes are observed was between 14.14 mm and 14.11 mm on the micrometer, which translates to 5.709 μm of actual carriage distance. Fringe diameters are so large near zero path difference because the interference pattern is a result of an superposition of wave crests, at near zero path difference the wave crests is small so the for two crests or troughs to align up takes a greater angle, or larger diameter fringes. You can see so many colours because the white light from the incandescent bulb contains all (or most) of the wavelengths of light, as certain parts of the light interfere with each other as the carriage is moves, other wavelengths construct resulting in the separation of colours contained within the white light. When the path length is zero, that means the path length is the same for all wavelengths and that is where the pattern appears nearly black. At large optical path differences the random separation of all the wavelengths and the non-exact alignment of the mirrors you cannot observe the vibrant colours except near zero path length differences.

III. CONCLUSION

First the interferometer was calibrated and the constant K was found, relating the micrometer movement to the actual carriage travel of the mirror and using a known source wavelength was found to be 0.1903 ± 0.0129, and this value was used throughout for the rest of our calculations.

Using the same method above with the measured K, we measured the wavelength of the green like spectral line of Hg, and found it to me 542.35 ± 53.0 nm, which only differed by 0.7% of the accepted value of 546.074 nm, which is also within out uncertainty estimates.

Similarly we calculated the separation of the Hg yellow lines. We measured the separation to be 2.124 ± 0.14 nm, which only differed by 0.9% compared to the accepted value of 2.106 nm.

Using a vacuum chamber and by varying the pressure (and by extension the number of particles in the chamber), we were able to measure the refractive index of air, n_{air}. Using the Lorenz-Lorentz law we were able to plot our results and calculate a number for n_{air} at STP, 760 mmHg at room temperature for Na yellow light to be 1.00034 ± 0.00013. This is consistent with theory as the refractive index of materials should be greater than 1 (the refractive index of a vacuum).