The Astronomical Telescope

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Section 1
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I. PURPOSE

The purpose of the experiment is observe and measure the magnification of compound lenses, in two special cases, the Astronomical Telescope which uses two positive lenses, and the Galilean Telescope which uses a positive and a negative lens. We will be measuring the magnification using several different methods such as the ratio of focal lengths, using direct observation, and by measuring and comparing the ratio of the entrance to exit pupil. We will also be investigating and measuring brightness and field of view as the size of the lens changes to try to measure the diameter of the pupil. We will also be measuring the angular resolving power of the astronomical telescope and the unaided eye and compare them with each other and theoretical expectations.

II. ANALYSIS

A. Measurement of Magnification

1. *Magnification from the Ratio of Focal Lengths* ($\frac{F}{f}$)

	Measurements (± 0.1 cm)	Average (± 0.06 cm)
F	19.166, 19.300, 19.200	19.222
f	4.626, 4.710, 4.744	4.693
Inter-lens Distance	23.9, 24.0, 24.0	24.0

TABLE I: Measurements and averages for the objective focal point F, the ocular focal point f, and the inter-lens distance between the objective and ocular when both focal points overlap

Sample Calculations for $\bar{F}$ using row 1 of Table I.

$$\bar{F} = \frac{\sum_i^n F_i}{n}$$
$$\bar{F} = \frac{19.166+19.300+19.200}{3}$$
$$\bar{F} = 19.222$$

Sample Calculations for $\Delta\bar{F}$

$$\Delta\bar{F} = \pm\frac{\Delta F}{\sqrt{n}}$$
$$\Delta\bar{F} = \pm\frac{0.1}{\sqrt{3}}$$
$$\Delta\bar{F} = \pm 0.06cm$$

Sample Calculations for $\frac{F}{f}$

$$\frac{F}{f} = \frac{19.222}{4.693}$$
$$\frac{F}{f} = 4.096$$

Sample Calculations for $\Delta \frac{F}{f}$

$$\Delta \frac{F}{f} = \sqrt{(\Delta F)^2 + (\Delta f)^2}$$
$$\Delta \frac{F}{f} = \sqrt{(0.06)^2 + (0.06)^2}$$
$$\Delta \frac{F}{f} = \pm 0.08$$

The magnification was calculated to be 4.096 ±0.08. When viewing the image though the eyepiece you can see a large, bright target compared to looking at the light source directly without the aide of the lenses. The inter-lens distance which was measured to be 24.0 ±0.06cm is equal to f+F = 19.222 + 4.963 = 23.915 ±0.08cm.

2. Magnification by Direct Observation ($\frac{L}{\ell}$)

We looked through the telescope at a wall with bricks. Using the unaided eye to look at the wall and looking through the telescope through the other, we observed 4 bricks in the space where one brick appears in the telescope image, meaning the magnification observed is 4 ±0.08. This coincides with the result from earlier with out measured magnification of 4.096 ±0.08, only deviating by 2.3%.

Sample Calculations for % deviation of observed magnification with $\frac{F}{f}$

$$\%_{deviation} = \frac{|4-4.096|}{4.096} \times 100\%$$
$$\%_{deviation} = 2.3\%$$

3. Magnification from the Ratio of Entrance to Exit Pupil ($\frac{D_o}{d}$)

	Measurements (± 0.1 cm)	Average (± 0.06 cm)
D_o	4.736, 4.776, 4.754	4.755
d	1.230, 1.246, 1.188	1.221
DE	6.480, 6.270, 6.412	6.387

TABLE II: Measurements and averages for the diameter of the objective lens D_o, the minimum diameter d (the exit pupil), and the the distance from the ocular lens to the exit pupil DE.

Sample Calculations for $\frac{D_o}{d}$

$$\frac{D_o}{d} = \frac{4.755}{1.221}$$
$$\frac{D_o}{d} = 3.894$$

Sample Calculations for $\Delta \frac{D_o}{d}$

$$\Delta \frac{D_o}{d} = \sqrt{(\Delta D_o)^2 + (\Delta d)^2}$$
$$\Delta \frac{D_o}{d} = \sqrt{(0.06)^2 + (0.06)^2}$$
$$\Delta \frac{D_o}{d} = \pm 0.08$$

Sample Calculations for % deviation of $\frac{D_o}{d}$ with $\frac{F}{f}$

$$\%_{deviation} = \frac{|3.894-4.096|}{4.096} \times 100\%$$
$$\%_{deviation} = 4.9\%$$

Sample Calculations for % deviation of $\frac{D_o}{d}$ with $\frac{L}{l}$

$$\%_{deviation} = \frac{|3.894-4|}{4} \times 100\%$$
$$\%_{deviation} = 2.7\%$$

The magnification by method of measuring the ratio to entrance to exit pupil ($\frac{D_o}{d}$) was measured to be 3.894 ±0.08 times magnification. This is also in line with our previous results only deviating 4.9% ad 2.7% for $\frac{F}{f}$ and $\frac{L}{l}$ respectively.

Sample Calculations for DE using measured values for F and f to be 19.222 ±0.06cm and 4.693 ±0.06cm respectively

$$DE = f \cdot (1 + \frac{f}{F})$$
$$DE = 4.693 \cdot (1 + \frac{4.693}{19.222})$$
$$DE = 5.839cm$$

Sample Calculations for % deviation of measured and calculated DE

$$\%_{deviation} = \frac{|5.839-6.387|}{6.387} \times 100\%$$
$$\%_{deviation} = 8.6\%$$

Our calculated and measured value for DE are similar, only differing by 8.6%, this is enough to confirm that the distance DE obeys the relation above.

B. Brightness of the Image

Using the diaphragm in front of the telescope to vary the light until a point where the image, while looking through the telescope, seems to get darker, we were able to find a diameter for the diaphragm $D_o$eff. I measured $D_o$eff for my eye to be 1.324 cm.

Sample Calculations for p, the diameter of my pupil using measured values for $D_o$eff of 1.324 cm and M to be 4.096

$$p = \frac{D_o eff}{M}$$
$$p = \frac{1.324}{4.096}$$
$$p = 0.323cm = 3.23mm$$

Sample Calculations for Δp

$$\Delta p = p \cdot \sqrt{(\frac{\Delta D_o eff}{D_o eff})^2 + (\frac{\Delta M}{M})^2}$$
$$\Delta p = 3.23mm \cdot \sqrt{(\frac{0.1cm}{1.324cm})^2 + (\frac{0.08}{4.096})^2}$$
$$\Delta p = \pm 0.25mm$$

My pupil diameter, p, was calculated to be 3.23 ±0.25mm which is on the order the size of what a pupil should be.

C. Field of View

D_e (± 0.1cm) : 3.774, 3.802, 3.796, 3.792
$\bar{D}_e$ = 3.791 (± 0.05cm)

Sample Calculations for $\bar{D_e}$.

$$\bar{D_e} = \frac{\sum_i^n D_{e_i}}{n}$$
$$\bar{D_e} = \frac{3.774+3.802+3.796+3.792}{4}$$
$$\bar{D_e} = 3.791$$

Sample Calculations for $\Delta\bar{D_e}$

$$\Delta\bar{D_e} = \pm\frac{\Delta D_e}{\sqrt{n}}$$
$$\Delta\bar{D_e} = \pm\frac{0.1}{\sqrt{4}}$$
$$\Delta\bar{D_e} = \pm 0.05cm$$

Distance to bricks from Objective: 671.5cm ±1cm
1 Brick = $\frac{81.2cm}{6}$ = 13.5 cm ±0.25cm

D_s	Number of Bricks in View	$\frac{\ell_b}{D_b}$	$\Delta\frac{\ell_b}{D_b}$	$\frac{D_s}{F}$	$\Delta\frac{D_s}{F}$	% deviation
2.196	6	0.121	0.000415	0.114	0.00520	6%
1.582	5	0.101	0.000401	0.082	0.00519	23%
1.370	4	0.080	0.000391	0.071	0.00519	13%
0.956	3	0.060	0.000383	0.050	0.00523	20%
0.682	2	0.040	0.000377	0.035	0.00513	14%

TABLE III: Measurements for the diameter of the stop D_s, the number of bricks observed, and the calculations for field of view, $\frac{\ell_b}{D_b}$ and $\frac{D_s}{F}$ which are the length of observed bricks divided by the distance to the bricks from the telescope and the diameter of the stop divide by the objective focal length respectively.

Sample Calculations for $\frac{\ell_b}{D_b}$ using Row 1 of Table III, and a conversion rate of 1 brick = 13.5 cm ±0.25cm and the distance to the bricks as 671.5cm ±1cm

$$\frac{\ell_b}{D_b} = \frac{6\times 13.5}{671.5}$$
$$\frac{\ell_b}{D_b} = 0.121$$

Sample Calculations for $\Delta\frac{\ell_b}{D_b}$ using Row 1 of Table III

$$\Delta\frac{\ell_b}{D_b} = \frac{\ell_b}{D_b}\cdot\sqrt{(\frac{\Delta\ell_b}{\ell_b})^2 + (\frac{\Delta D_b}{D_b})^2}$$
$$\Delta\frac{\ell_b}{D_b} = 0.121\cdot\sqrt{(\frac{0.25}{6\times 13.5})^2 + (\frac{1}{671.5})^2}$$
$$\Delta\frac{\ell_b}{D_b} = 0.000415$$

Sample Calculations for $\frac{D_s}{F}$ using Row 1 of Table 3

$$\frac{D_s}{F} = \frac{2.196}{19.222}$$
$$\frac{D_s}{F} = 0.114$$

Sample Calculations for $\Delta\frac{D_s}{F}$ using Row 1 of Table III

$$\Delta\frac{D_s}{F} = \frac{D_s}{F} \cdot \sqrt{(\frac{\Delta D_s}{D_s})^2 + (\frac{\Delta F}{F})^2}$$
$$\Delta\frac{D_s}{F} = 0.114 \cdot \sqrt{(\frac{0.1}{2.196})^2 + (\frac{0.06}{19.222})^2}$$
$$\Delta\frac{D_s}{F} = 0.00520$$

Sample Calculations for % deviation of measured and calculated Field of View using Row 1 of Table III

$$\%_{deviation} = \frac{|0.121 - 0.114|}{0.114} \times 100\%$$
$$\%_{deviation} = 6\%$$

The measured Field of View with the calculated theoretical field of view line up rather nicely, are are close to the margin of the estimated uncertainty. The measured and calculated deviated by only 6-23% in our 5 trials.

D. Resolving Power

Hole Separation (mm)	$D_{telescope}$ ($\pm$ 1cm)	$\alpha_{measured}$ (rad)	% deviation from $\alpha_{theoretical}$
1	525	1.90×10^{-4}	1200%
2	—	—	—
3	—	—	—

TABLE IV: Measurements for the distance at which two holes separated by some distance were no longer distinguishable as two separate holes when viewed through the telescope using an objective Diameter $D_o = 4.755 \pm 0.06$cm. The lab was not large enough to test the resolving power for the hole separation greater than 1mm.

Hole Separation (mm)	$D_{unaided}$ ($\pm$ 1cm)	$\alpha_{measured}$ (rad)	% deviation from $\alpha_{theoretical}$
1	210	4.76×10^{-4}	130%
2	330	6.06×10^{-4}	200%
3	—	—	—

TABLE V: Measurements for the distance at which two holes separated by some distance were no longer distinguishable as two separate holes when viewed through the unaided eye using an objective Diameter p = 3.23 $\pm$0.25mm. The lab was not large enough to test the resolving power for the hole separation greater than 2mm.

Sample Calculations for $\alpha_{measured}$ using Row 1 of Table IV

$$\alpha_{measured} = 2 \cdot \arctan\frac{\frac{separation}{2}}{D_{telescope}}$$
$$\alpha_{measured} = 2 \cdot \arctan\frac{0.05}{525}$$
$$\alpha_{measured} = 1.90 \times 10^{-4} rad$$

Sample Calculations for $\alpha_{theoretical}$ for the telescope using a wavelength of 550nm and D_o = 4.755cm

$$\alpha_{theoretical} = 1.22 \cdot \frac{\lambda}{D_o}$$
$$\alpha_{theoretical} = 1.22 \cdot \frac{5.50 \times 10^{-7}}{0.04755}$$
$$\alpha_{theoretical} = 1.41 \times 10^{-5} rad$$

Sample Calculations for $\alpha_{theoretical}$ for the unaided eye using a wavelength of 550nm and p = 3.23mm

$$\alpha_{theoretical} = 1.22 \cdot \frac{\lambda}{p}$$
$$\alpha_{theoretical} = 1.22 \cdot \frac{5.50\times10^{-7}}{3.23\times10^{-3}}$$
$$\alpha_{theoretical} = 2.08 \times 10^{-4} rad$$

Sample Calculations for % deviation of measured and calculated α

$$\%_{deviation} = \frac{|1.90\times10^{-4} - 1.41\times10^{-5}|}{1.41\times10^{-5}} \times 100\%$$
$$\%_{deviation} = 1200\%$$

There were large discrepancies between the measured and theoretical angular resolving power for both the telescope and the unaided eye. With the telescope angular resolving power deviating by 1200% and the unaided eye deviating by 130% for the 1mm case, and the unaided eye deviating by 200% for the 2mm case. This may be due to the subjective nature of the test, and as the pupil of a human's eye will contract and dilate naturally, the parameter calculating the theoretical angular resolving power at the time of the test was most likely different.

E. Galilean Telescope

1. Determining the Focal Length of the negative lens

The negative lens was placed 15cm away from the objective and the screen was placed at a point on the other side of the lens such that the image S_2' was focussed on the screen. The distance to the screen from the objective was measured to be 22.9 $\pm$0.1cm.

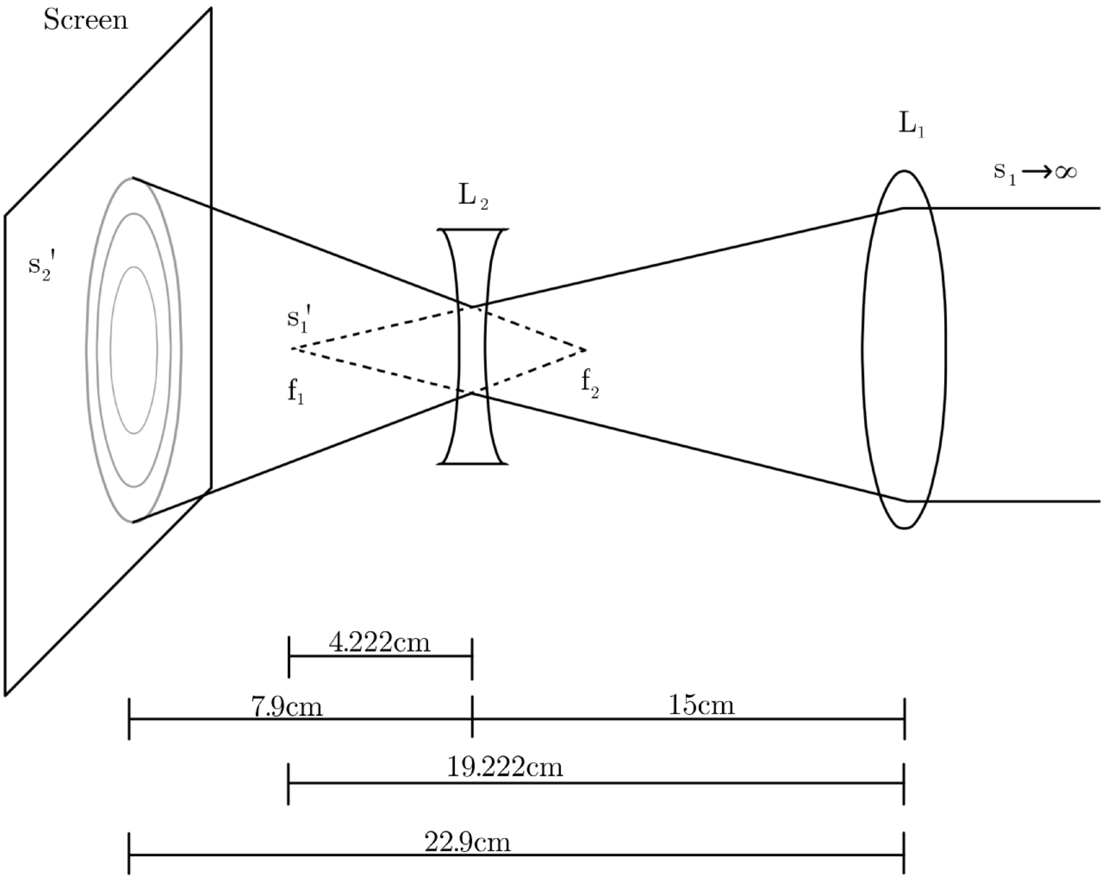

FIG. 1: Diagram of the Galilean telescope (not to scale) with relevant points and distances labelled.

Sample Calculations for f_2 using the thin lens law

$$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s'_2}$$
$$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{-s'_1}$$
$$\frac{1}{f_2} = \frac{1}{7.9cm} + \frac{1}{-(19.222cm-15cm)}$$
$$\frac{1}{f_2} = \frac{1}{7.9cm} + \frac{1}{-(4.222)}$$
$$f_2 = -9.07cm$$

Sample Calculations for Δf_2

$$\Delta f_2 = \sqrt{(0.06)^2 + (0.1)^2 + (0.1)^2 + (0.1)^2}$$
$$\Delta f_2 = \pm 0.062cm$$

2. Magnification by the Ratio of Focal Lengths

Measured Inter-lens distance when focussed image is seen: 8.56 ±0.1cm

The Inter-lens distance still seems to follow the relationship of $f_1 + f_2$, but in this case f_2 is negative so the inter-lens distance is equal to 19.222cm-9.07cm = 10.15cm

Sample Calculations for M using f_1 = 19.222cm and f_2 = -9.07cm

$$M = \frac{-f_1}{f_2}$$
$$M = \frac{19.222}{9.07}$$
$$M = 2.1$$

Sample Calculations for ΔM

$$\Delta M = 2.1 \cdot \sqrt{(\frac{0.06}{19.222})^2 + (\frac{0.062}{9.07})^2}$$
$$\Delta M = \pm 0.016$$

The image of a distant object produced by the Galilean telescope is not inverted in the case of the Astronomical telescope.

3. Measurement of Magnification

Using direct observation on a brick wall, we were able to observe the magnification to be about 2 times which is in line with the calculated magnification of 2.1±0.016.

We could not employ the method of magnification calculation using the ratio of the entrance to exit pupil because the Galilean telescope produces a virtual image which cannot be measured.

4. Comparisons between the Astronomical and Galilean Telescopes

The length of the Galilean telescope is shorter than the length of the Astronomical telescope due to the negative focal length of the bi-concave lens.

The position of the exit pupil for the astronomical telescope is always centred because the image it produces is real. However the Galilean telescope produces a virtual image in the on the opposite side of the lens. The exit pupil can be off centred by some degree when not viewing an object directly centred in the field of view of the telescope.

The field of view of the Galilean telescope however, is smaller than the Astronomical telescope which gives the astronomical telescope an advantage in this regard.

The magnification for both cases is the ratio of the focal lengths of the lenses used.

III. CONCLUSION

We were able to observe the magnification properties of the Astronomical and Galilean telescopes using several different methods. Using the ratio of the focal lengths, we were able to calculate the Magnification of the Astronomical telescope to be about 4.096 ± 0.08, and the Galilean telescope to be about 2.1 ± 0.016. We also employed direct observation and found the magnification factor to be about 4 and 2 for the Astronomical and Galilean telescopes respectively, which were in line with our calculated values. We used a third method, by comparing the ratio of the entrance to exit pupil and were able to measure the magnification for the Astronomical telescope to be 3.894 ± 0.08, which only deviated from the previously calculated value by only 4.9% and was close to the margins on uncertainty. The method of comparing the ratio of the entrance to exit pupil could not be employed for the Galilean telescope because of the image it produces is virtual, and therefore could not be measured.

Using a stop, we were able to vary the effective diameter of the objective lens and were able to measure and calculate the size of my pupil. We calculated a value of 3.23 ± 0.25mm for the size of my pupil which is on the order of a regular pupil size.

We also investigated the field of view of the astronomical telescope by using bricks. The field of view measurements were in line with our expected theoretical values for the varied effective objective diameter. Our measured values for the field of view only deviated by 6-23% from the theoretical values.

We took the time to test out the resolving power of the Astronomical telescope and the unaided eye and compare those values with theoretical. We found that our values varied quite a bit with the theoretical limits. Our measured values deviated by 1200% for the Astronomical telescope, and 130-200% for the unaided eye. However due to the subjective nature of this test it is difficult to come up with conclusive measurements, especially with the unaided eye as the pupil constantly dilates and contracts between measurements. The size of the lab was also not optimal in finding a distance within the lab at which 2 objects could no longer be resolved. Chromatic aberration and the assumption that the light we were viewing was that of a fixed wavelength also incurred significant error in our comparisons as it is unrealistic to our scenario.

We were able to measure the focal length of the negative lens in the Galilean telescope and found it to be -9.07 ± 0.062cm

We were able to discern some advantages and disadvantages between the Astronomical and Galilean telescopes. The Galilean telescope has a shorter length than the Astronomical telescope due to its negative focal length. The position of the exit pupil on the Astronomical telescope is fixed which the Galilean can be off centred when not viewing an object head on with the telescope. The field of view is also smaller as a result for the Galilean telescope. The magnification for both telescopes followed the same property, of being proportional $\frac{F}{f}$.

Uncertainty in our measurements were largely limited to the mounting equipment. Such as the mount which held the lenses and trying to measure distances between lenses, there were no conclusive start and end points which we could measure between, so an uncertainty of 0.1cm was assumed. When measuring longer distances using several meter sticks, between an object and the telescope, the start of the telescope could not be accurately compared to the values on the meter stick so a larger uncertainty of 1cm was used for these measurements.