Dispersion of Glass by the Method of Minimum Deviation

Andy Chmilenko, 20310799
Instructor: Jeff Gardiner
Section 1
(Dated: 2:30 pm Tuesday September 24, 2013)

I. PURPOSE

The Purpose of this experiment is to determine the indices of refraction of the glass prism at different wavelengths; thence to determine the dispersion curve for that glass and compare it with Cauchy's equation.

II. ANALYSIS

A. Determining the Apex Angle

$\alpha_L \pm 30''$	$\alpha_R \pm 30''$	$\alpha \pm 42''$
$119°57'$	$240°8'$	$60°5'$
$87°16'$	$207°25'$	$60°4'$
$71°9'$	$191°23'$	$60°7'$

TABLE I: Measured angles from reflected collimated light off of the prism from the left (α_L) and right(α_R) sides, and the calculated apex angle (α)

Sample Calculations for α using Table I row 1

$$\alpha = \frac{|\alpha_L - \alpha_R|}{2}$$
$$\alpha = \frac{|119°57' - 240°8'|}{2}$$
$$\alpha = \frac{120°11'}{2}$$
$$\alpha = 60°5'30''$$

Sample Calculations for $\Delta\alpha$

$$\Delta\alpha = \sqrt{(\Delta\alpha_L)^2 + (\Delta\alpha_R)^2}$$
$$\Delta\alpha = \sqrt{(30'')^2 + (30'')^2}$$
$$\Delta\alpha = \sqrt{(0.25'')^2 + (0.25'')^2}$$
$$\Delta\alpha = 42.43'' = 42''$$

Sample Calculations for $\bar{\alpha}$

$$\bar{\alpha} = \frac{\sum_i^n \alpha_i}{n}$$
$$\bar{\alpha} = \frac{60°5' + 60°4' + 60°7'}{3}$$
$$\bar{\alpha} = 60°5'20''$$

The apex angle for the prism, α was measured to be $60°5' \pm 42''$.

B. Determining the Refractive Index

Colour	Wave-length in Air ($\AA$)	Setting of Minimum Deviation Left $\pm 30''$	Setting of Minimum Deviation Right $\pm 30''$	Angle of Minimum Deviation δ_m $\pm 42''$	Refractive Index, n	Δn ($\times$ 19^{-4})
Red - H	6563	$132°1'$	$227°36'$	$47°47'$	1.615	± 2.445
Yellow III - Hg	5790	$131°29'$	$228°11'$	$48°21'$	1.620	± 2.437
Yellow I - Hg	5770	$131°28'$	$228°14'$	$48°23'$	1.621	± 2.437
Green - Hg	5461	$131°7'$	$228°30'$	$48°41'$	1.624	± 2.432
Blue-Green - H	4861	$130°23'$	$229°17'$	$49°27'$	1.631	± 2.422
Blue - Hg	4358	$129°22'$	$230°19'$	$50°29'$	1.642	± 2.407
Violet - Hg	4078	$128°34'$	$231°4'$	$51°15'$	1.649	± 2.396
Violet - Hg	4047	$128°29'$	$231°6'$	$51°19'$	1.650	± 2.395

TABLE II: Measured angles from diffracted collimated light for different wavelengths, produced by a Hg lamp and a H lamp.

Sample Calculations for δ_m using Table II row 1

$$2\delta_m = |L - R|$$
$$\delta_m = \frac{|L-R|}{2}$$
$$\delta_m = \frac{|132°1' - 227°36'|}{2}$$
$$\delta_m = \frac{95°35'}{2}$$
$$\delta_m = 47°47'30''$$

Sample Calculations for $\Delta\delta_m$

$$\Delta\delta_m = \sqrt{(\Delta L)^2 + (\Delta R)^2}$$
$$\Delta\delta_m = \sqrt{(30'')^2 + (30'')^2}$$
$$\Delta\delta_m = \sqrt{(0.25'')^2 + (0.25'')^2}$$
$$\Delta\delta_m = 42.43'' = 42''$$

Sample Calculations for n using Table II row 1 assuming $n = 1$

$$\frac{n'}{n} = \frac{\sin\dfrac{\alpha + \delta_m}{2}}{\sin\dfrac{\alpha}{2}}$$
$$n' = \frac{\sin\dfrac{60°5' + 47°47'}{2}}{\sin\dfrac{60°5'}{2}}$$
$$n' = 1.615$$

Sample Calculations for Δn using Table II row 1

$$\Delta n = \sqrt{\begin{aligned}(\cos\frac{\alpha+\delta_m}{2}\cdot\sqrt{(\Delta\alpha)^2+(\Delta\delta_m)^2})^2 \\ +(\cos\frac{\alpha}{2}\cdot\Delta\alpha)^2\end{aligned}}$$

$$\Delta n = \sqrt{\begin{aligned}(\cos\frac{60°5' + 47°46'}{2}\cdot\sqrt{(42'')^2+(42'')^2})^2 \\ +(\cos\frac{60°5'}{2}\cdot 42'')^2\end{aligned}}$$

$$\Delta n = 2.445\times 19^{-4}$$

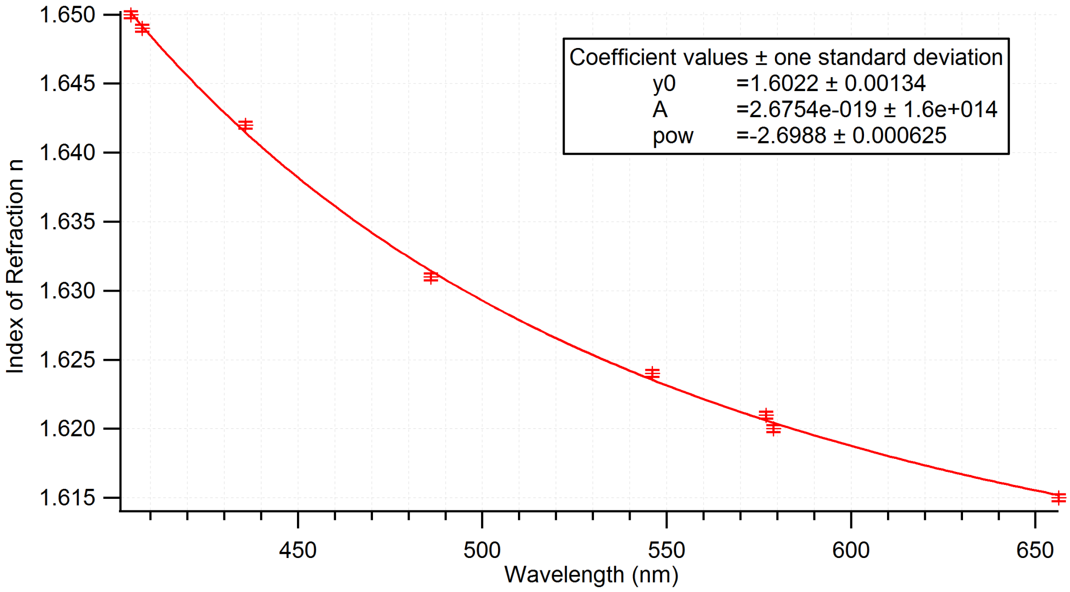

FIG. 1: Calculated index of refraction from minimum deviation measurements plotted versus wavelength.

This means the glass at any given wavelength is defined as $\frac{dn}{d\lambda}$ of the fit curve in Fig.1:

$$\frac{dn}{d\lambda} = -2.699\cdot 2.675\times 10^{-19}\cdot\lambda^{-3.699} \tag{1}$$

Using this equation we can find the dispersion of the glass prism at a wavelength of λ = 500nm, red light.

Sample Calculations for dispersion of the glass prism at a wavelength of λ = 500nm

$$\frac{dn}{d\lambda} = -2.699\cdot 2.675\times 10^{-19}\cdot(500nm)^{-3.699}$$
$$\frac{dn}{d\lambda} = -1.466\times 10^{5}$$

Using the plot to find indices of refraction for the F, C, and D Fraunhofer lines (wavelengths 4861, 6563, 5893 $\mathring{A}$ respectively), we can calculate the dispersive power, ω, of the glass of the prism using:

$$\omega = \frac{n_F - n_C}{n_D - 1} \tag{2}$$

Sample Calculations for dispersive power, ω

$$\omega = \frac{n_F - n_C}{n_D - 1}$$
$$\omega \approx \frac{1.634 - 1.616}{1.618 - 1}$$
$$\omega \approx 2.913 \times 10^{-2}$$

Comparing our calculated value of $\omega \approx 2.913 \times 10^{-2}$ to the values in Appendix B of the lab manual, Dense Flint is the closest material with a referenced value of .0295 for ω.

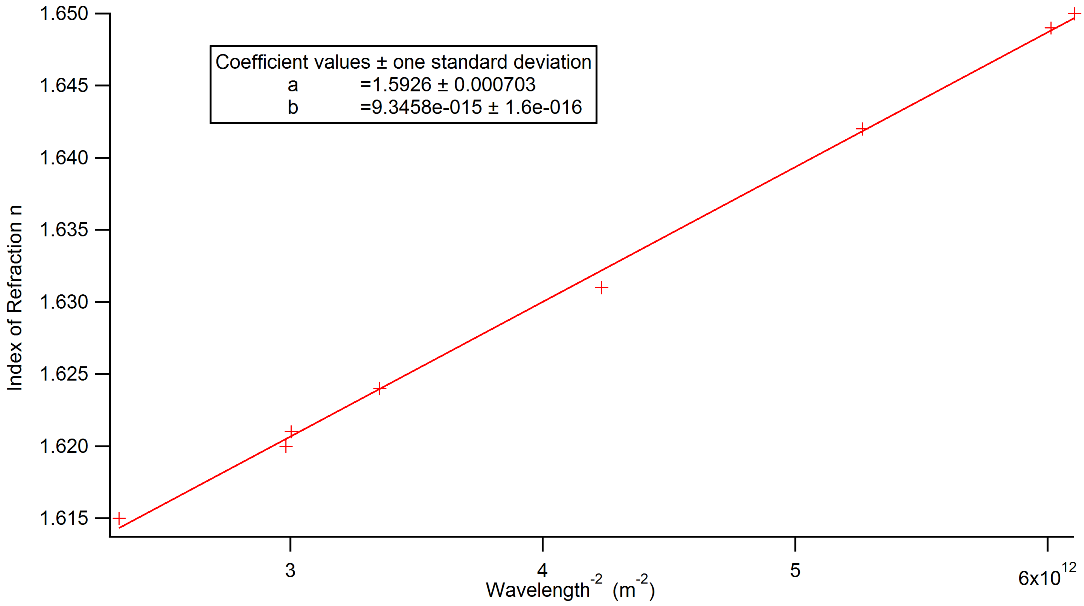

FIG. 2: Calculated index of refraction from minimum deviation measurements plotted versus inverse square of the wavelength ($\frac{1}{\lambda^2}$).

By plotting index of refraction over the inverse of the square wavelength, we can determine the constants A and B of Cauchy's Equation. Experimentally we got A = 1.593 and B = 9.346×10^{-15}:

$$n(\lambda) = 1.593 + 9.346 \times 10^{-15} \cdot \frac{1}{\lambda^2} \tag{3}$$

and,

$$\frac{dn(\lambda)}{d\lambda} = (-2) \cdot 9.346 \times 10^{-15} \cdot \frac{1}{\lambda^3} \tag{4}$$

Calculating n using some wavelength and Eq.3, we can compare this value to our best fit line from Fig.1.

Sample Calculations for n for λ = 460nm, using Eq.3

$$n(\lambda) = 1.593 + 9.346 \times 10^{-15} \cdot \frac{1}{460nm^2}$$
$$n(\lambda) = 1.637$$

Comparing this by inspecting Fig.1, we read a value of ≈ 1.637 as well, which fits our predictions. Comparing the dispersion at 500nm with our previous calculation we get a value of -1.495×10^5, compared to our previously calculated value of -1.466×10^5, only deviated 2

Sample Calculations for n for $\frac{dn(\lambda)}{d\lambda}$ at 500nm, using Eq.4

$$\frac{dn(\lambda)}{d\lambda} = (-2) \cdot 9.346 \times 10^{-15} \cdot \frac{1}{500nm^3}$$
$$\frac{dn(\lambda)}{d\lambda} = -1.495 \times 10^5$$

Sample Calculations for % deviation of calculated dispersion values for light at 500nm

$$\%_{deviation} = \frac{|1.495-1.466|}{1.495} \times 100\%$$
$$\%_{deviation} = 1.9\%$$

III. CONCLUSION

We were able to determine the apex angle of the prism to be to be $60°5' \pm 42''$. Uncertainty in our measurements was taken to be half of the smallest deviation of the measuring apparatus which was $30''$. We also measured various angles of minimum deviation for several different wavelengths using a Hydrogen lamp and a Mercury lamp. From these measurements we were able to calculate various indices of refraction for the given wavelengths, we calculated indices of refraction around 1.6 for the range of wavelengths we could observe, as seen in Table II. Uncertainty incurred is also from the limit of the smallest deviation of the measuring apparatus.

From this we were able to plot our results and calculate indices of refraction for various wavelength, as well as the dispersion for the glass prism we were using. We calculated the dispersion of red light at 500nm to be approximately -1.466$\times 10^5$. We were also able to calculate the dispersive power of the glass of the prism, and found it to be $\omega \approx 2.913 \times 10^{-2}$, and looking up this value in to the Appendix B of the lab manual concluded that the material the prism is made of is most likely Dense Flint with a referenced value of .0295 for ω.

By plotting n versus $\frac{1}{\lambda^2}$, were were able to experimentally determine the constants A and B for Cauchy's Equation, to be 1.593 and 9.346$\times 10^{-15}$ respectively. Using these constants and Cauchy's Equation we were able to calculate a refractive index with the equation, for example 460nm we calculated a refractive index of 1.637, comparing this with our experimental plot, 460nm has a refractive index of 1.637 as well which fits our predictions. Similarly we could also calculate the dispersion of the glass at red light of 500nm using Cauchy's equation to be -1.495$\times 10^5$, compared to our previously calculated value only deviated by 2