Assignment 2

Due Feb 10, 2015 at the beginning of lecture. Marked out of 30 and worth 10% of your final mark.

1. **[0 marks]:** Read Chapter 5,14. Do problems: 5.6,5.7, 5.8, 14.1,14.2. (Note: these will not be graded.)

2. **[12 marks]:** The Planck function B_λ describes the intensity of a blackbody emission, per unit wavelength:

$$B_\lambda = \frac{2hc^2}{\lambda^5}\left(e^{\frac{hc}{\lambda kT}} - 1\right)^{-1}. \tag{1}$$

   a) (3 marks) Make a graph of B_λ vs λ, for T=5500K, over the wavelength range λ = 100nm–2μm. Calculate the wavelength where this function reaches a maximum ($\lambda_{\rm peak}$) and verify that it agrees with your graph.

   b) (2 marks) Now make a graph of B_ν vs ν, for $T = 5500$K and $(0.15–30)\times10^{14}$Hz. Calculate the frequency where this function peaks ($\nu_{\rm peak}$). What wavelength does this correspond to?

   c) (2 marks) Which one is correct? They both are, of course. The difference comes because of the choice to divide up the energy in terms of equal bins of wavelength, or equal bins of frequency. Show that $|d\lambda| \neq |d\nu|$, but $|d\ln\lambda| = |d\ln\nu|$. Show that the function that describes the radiated power per unit area, per unit solid angle, and per unit $\ln\lambda$ (or $\ln\nu$) is $\lambda B_\lambda = \nu B_\nu$.

   d) (2 marks) From this, derive an expression for $\lambda_{\rm peak}$, the value of λ for which νB_ν=λB_λ is a maximum.

   e) (3 marks) Make a graph of $\log_{10}(\lambda B_\lambda) = \log_{10}(\nu B_\nu)$ as a function of $\log_{10}(\lambda)$ for T = 3000K, T = 5500K and T = 30000K. Show one graph over a wide wavelength range, and another spanning just the range of visible light (400nm–800nm). By how much does $\log(\lambda B_\lambda)$ vary over visible wavelengths, for each star? What can you conclude about the "colour" of the Sun over this wavelength range?

3. **[3 marks]:** Consider a box of electrically neutral hydrogen gas that is maintained at a constant volume V. In this simple situation, the number of free electrons must equal the number of HII ions: $n_eV = N^{II}$. Also, the total number of hydrogen atoms (both neutral and ionized), $N_t = N^I + N^{II}$, is related to the density of the gas by $N_t = \rho V/(m_p + m_e) \approx \rho V/m_p$, where m_p is the mass of the proton. Use these expressions, and the Saha equation, to derive a quadratic equation for the fraction of ionized atoms, as a function of T.

4. **[15 marks]:** A useful approximation for a stellar atmosphere is the *Eddington approximation.* The main assumption is that the radiative flux is constant (and thus equal to the surface flux, $\sigma_{\rm SB}T_e^4$), and that the necessarily anisotropic radiation field (integrated over all wavelengths) is described by

$$I = \begin{cases} I_{\rm out}, 0 < \theta < \pi/2 \\ I_{\rm in}, \pi/2 < \theta < \pi, \end{cases} \tag{2}$$

where $I_{\rm in} \neq I_{\rm out}$ are constants. With a bit of work it can be shown that this leads to a predicted temperature gradient:

$$T^4 = \frac{3}{4}T_e^4\left(\tau + \frac{2}{3}\right), \tag{3}$$

where τ is the optical depth.

a) (3 marks) Consider a typical A0V star, with $T_{\rm e} = 10,000K$, photosphere density $\rho = 10^{-6}{\rm kg/m^3}$, and mean opacity $\kappa = 3.0{\rm m^2/kg}$. Make a graph of T (in K) as a function of s (in km), over the range $0 < s < 1000$km. At what depth in the atmosphere does T=T_e? At what depth does $\tau = 2/3$? Give your answers in both km and in units of the solar radius, $R_\odot$. If we define the surface as the point where $\tau = 0$, what is the surface temperature, in terms of T_e?

b) (4 marks) Assume the photosphere is electrically neutral and made up entirely of hydrogen. Calculate an expression for f_2, the fraction of hydrogen atoms in the first excited state ($n = 2$) as a function of temperature. (*Hint: Recall your answer from question 3, above.*). Using the same stellar parameters as in part b), make a graph of $f_2(s)$ as a function of s, over the range $0 < s < 1000$km. At what radius or range of radii (in km) is this fraction largest?

c) (5 marks) The opacity of photons with wavelengths corresponding to the Balmer series (e.g. an Hα photon with $\lambda = 656.3$nm) will be much higher than $\kappa = 3.0{\rm m^2/kg}$, because they are more likely to interact with hydrogen atoms in the $n = 2$ level. Assume the opacity of these photons, interacting with a gas of hydrogen in this state, is $\kappa_{\rm Balmer} = 3.5 \times 10^5$ ${\rm m^2/kg}$. This is in addition to the opacity they see from hydrogen in all other states ($\kappa = 3.0{\rm m^2/kg}$). Use this to calculate the optical depth of Balmer photons as a function of physical depth in the atmosphere. Make a plot of τ as a function of physical depth for both Balmer photons and non-Balmer photons of similar frequency, on the same graph. Plot over a range $0 < \tau < 1$, and choose a suitable corresponding x-axis range. Using this, what is the physical depth for which the total optical depth of a Balmer photon is $\tau_{\rm Balmer} = 2/3$?

d) (3 marks) How do you expect the observed flux of radiation at this wavelength ($\lambda = 656.3$nm) to compare with the flux of radiation at a nearby, but non-Balmer wavelength? Will it be comparable, brighter, or fainter, and by how much?